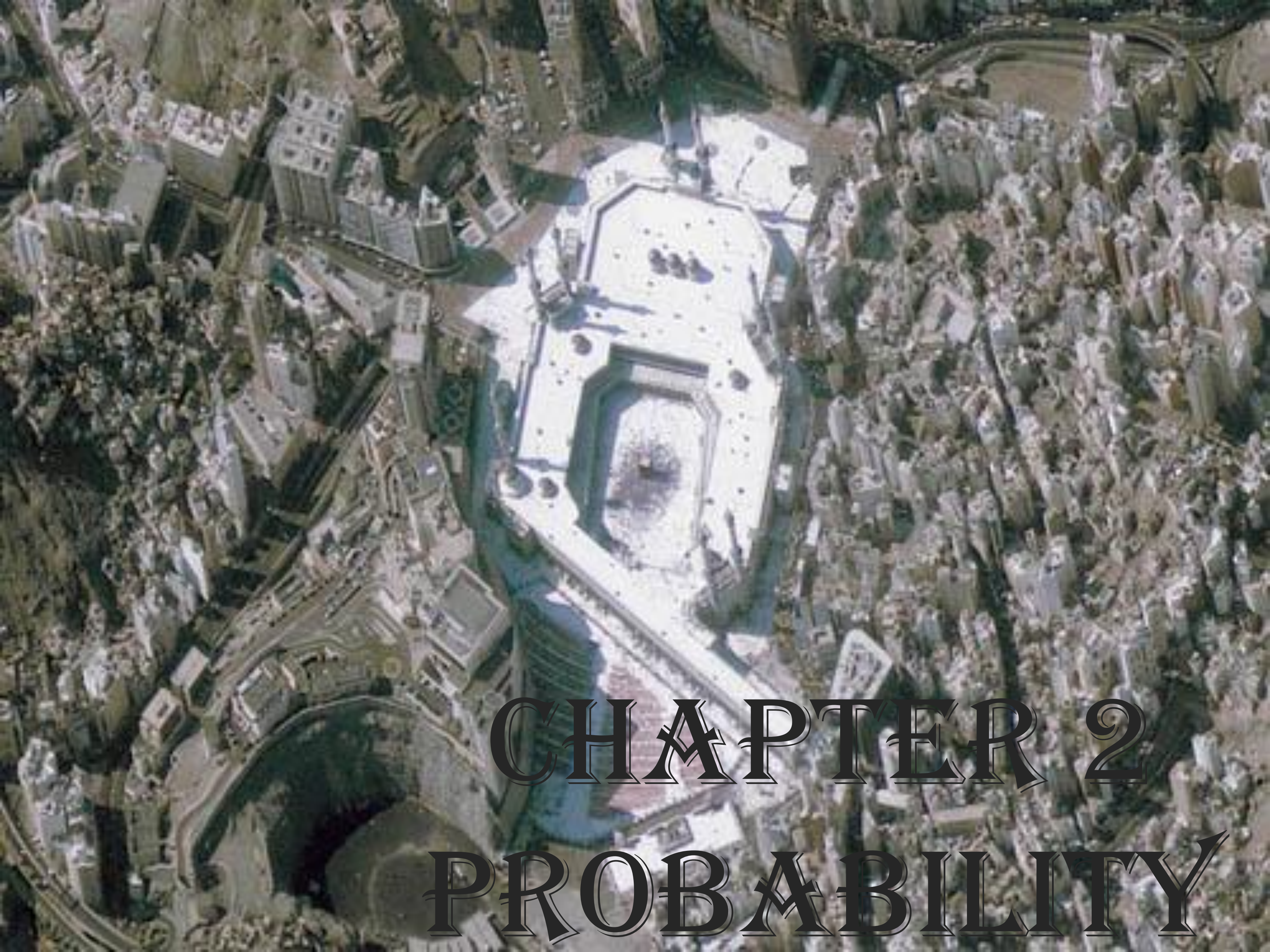
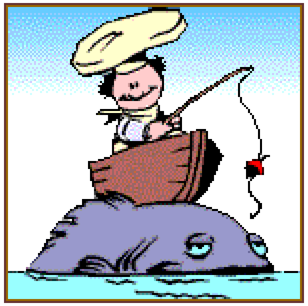


Chapter 2
Probability

Learning Objectives

1. Understand and describe sample space and events for random experiments
2. Interpret and use probabilities of outcome
3. Interpret and calculate conditional probabilities of events
4. Determine the independence of events and used it to calculate probabilities
5. Use Bayes' theorem to calculate conditional probabilities

Life is full of uncertainty



- It can be impossible to say what will happen from one minute to the next, but
- Probability lets you predict the future → helps to make informed decisions

Probabilities…

- The probability of any outcome of a random phenomenon is the proportion of times the outcome would occur in a very long series of repetitions.
- List the outcomes of a random experiment…

General way to say

$$P(A) = \frac{n(A)}{n(S)}$$

Probability of event A occurring → $P(A)$

Number of ways of getting an event A → $n(A)$

The number of possible outcomes → $n(S)$

Classical Approach…

- If an experiment has n possible outcomes, this method would assign a probability of 1/n to each outcome.

Example 1

Experiment: Rolling a ***die***

Sample Space: S = {1, 2, 3, 4, 5, 6}

Probabilities: Each sample point has a 1/6 chance of occurring.

Equally Likely Outcomes

- Probabilities under equally likely outcome case is simply the number of outcomes making up the event, divided by the number of outcomes in S.

Example 2

A die toss, A={2, 4, 6}, so

P(A) = 3/ 6 = 1/2 = .5

Example 3

Coin toss, A={H}, P(A) = 1/2= .5

Sample Space & Event

- Set of all possible outcomes is called the sample space, S.

  <u>Example 4</u>

  Die toss: S={1, 2, 3, 4, 5, 6}

  Toss coin twice: S={HH, TT, HT, TH}
- An individual outcome of a sample space is called a simple event

→ Discrete $\Rightarrow$ Have finite an element

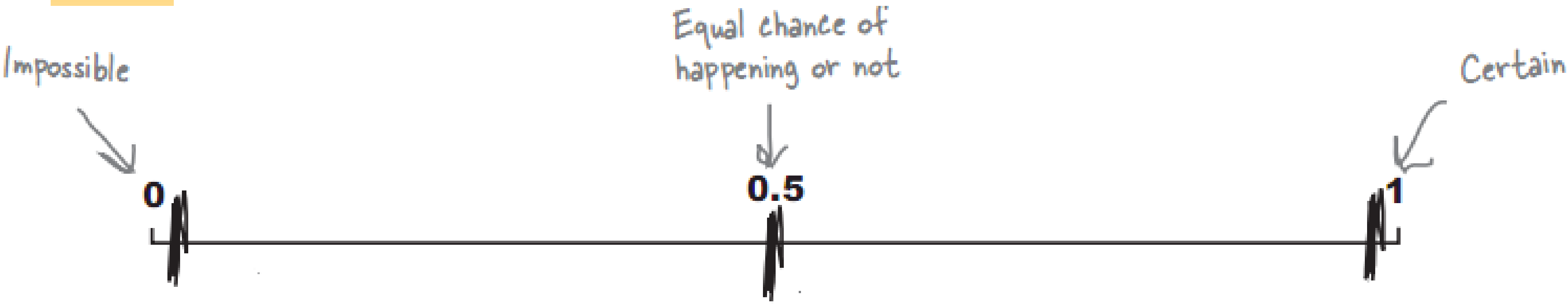
→ Continue $\Rightarrow$ Have an interval number

Events

→ is a collection or set of one or more simple events in a sample space

→ An outcome or occurrence that has a probability assigned to it

→ Illustration :



Example 5

Roll of a die: S = {1, 2, 3, 4, 5, 6}

Simple event: the number "3" will be rolled

Event: an even number (one of 2, 4, or 6) will be rolled

Discrete Sample Space

Example 6

- Eksperiment : Flip one dice
- Sample Space : S = {1,2,3,4,5,6}
- Event :

  A = Odd number = {1,3,5}

  B = Even number = {2,4,6}

Example 7

- Eksperiment : Flip two coins
- S = {MM, MB, BM, BB}
- Event :

  A = Both of side is same = {MM, BB}

  B = at least 1 M= {MM, MB, BM}

Continue Sample Space

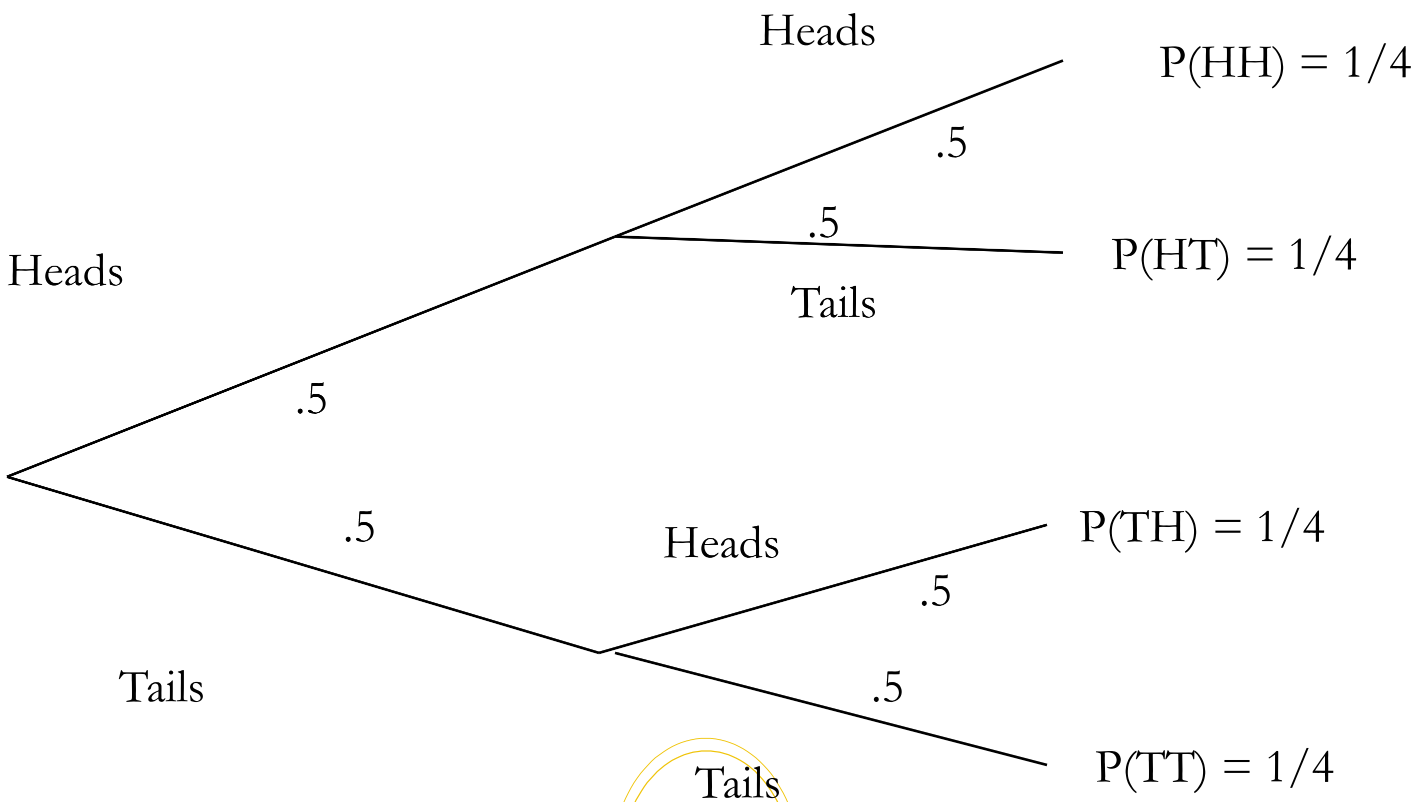
Example 8

- Eksperiment : Recording an IPK of students
- Outcome : Real number between 0 and 4
- $S = \{x \in R: 0 \leq x \leq 4\}$
- Event :

  A = IPK more than 3 = $\{3 < x \leq 4\}$

  B = IPK bellow 2 = $\{0 \leq x < 2\}$

Example 9 Probability Trees : Flip a coin



Exhaustive & Mutually Exclusive

- This list must be ***exhaustive***, i.e. ALL possible outcomes included.

<u>Ex10</u>: Die roll ~~{1,2,3,4,5}~~ → Die roll {1,2,3,4,5,6}

- The list must be ***mutually exclusive***, i.e. no two outcomes can occur at the same time:

<u>Ex11</u> : Die roll ~~{odd number or even number}~~

Die roll{ number less than 4 or even number}

Venn diagram

Relationship between an events with sample space can describe with Venn diagram

COMPLEMENTARY EVENTS (A’)

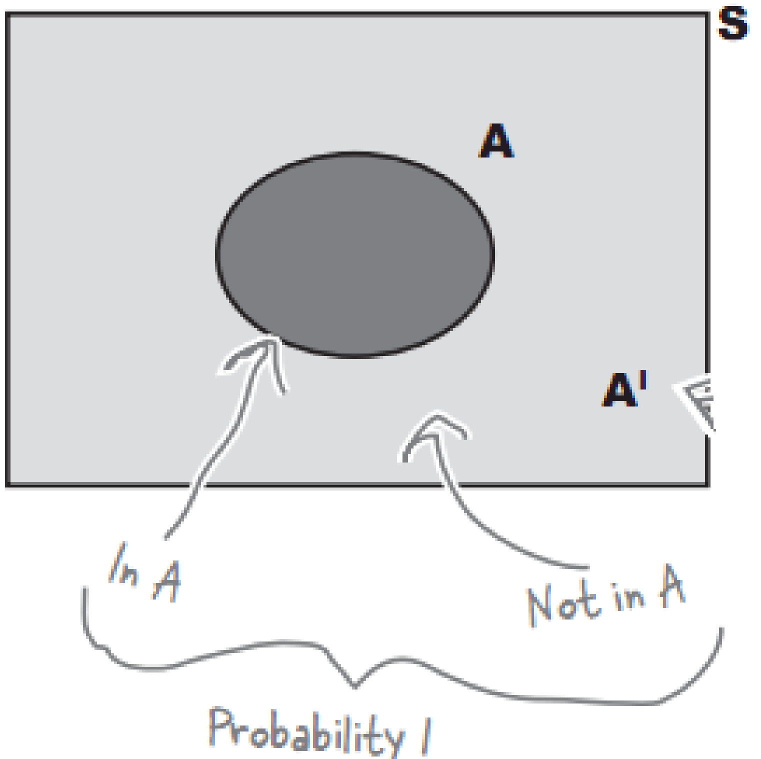
→ Indicating the event that A does not occur

Example 12

S : Natural number

A : Odd number

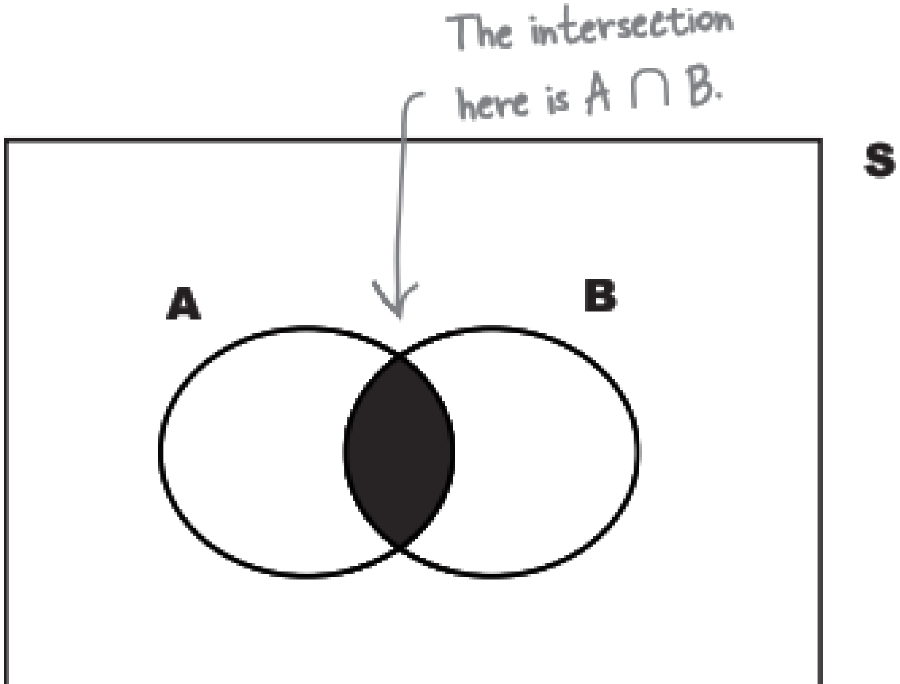
A’ : Even number



Intersection

- **Intersection between an event** A and B is an events that comprise an intersection event A and B

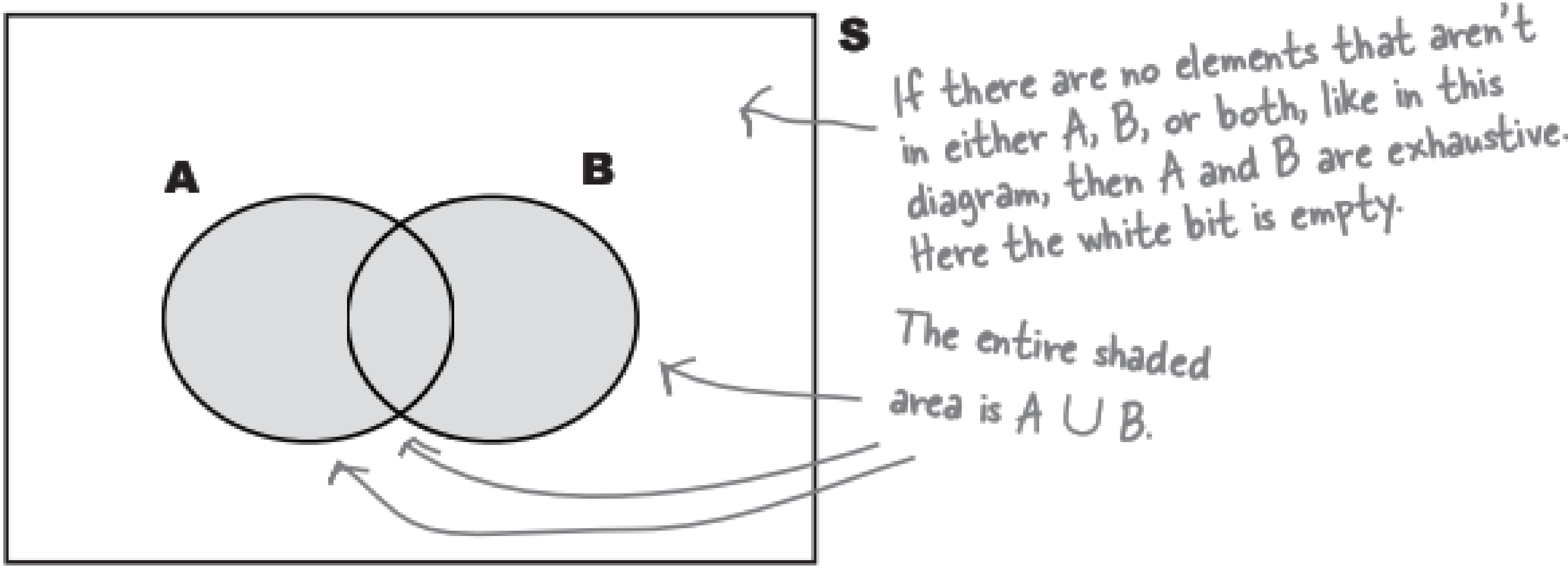
$$\text{“A and B”} : A \cap B = \{\omega \in \Omega \mid \omega \in A \text{ dan } \omega \in B\}$$



Union

- Union two event A and B notated with A U B
- Is an events that covers all of member A either B or both of

“A or B” : $A \cup B = \{\omega \in \Omega \mid \omega \in A \text{ or } \omega \in B\}$



Mutually Exclusive Events

- Two event A and B will be mutually exclusive events if both of A and B disjoint each other or $A \cap B = \emptyset$
- Mean : no intersection between A and B

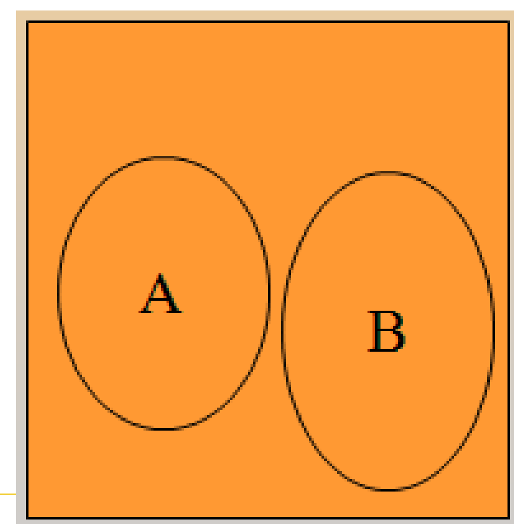
There's a difference between exclusive and exhaustive.

If events A and B are exclusive, then

$$P(A \cap B) = 0$$

If events A and B are exhaustive, then

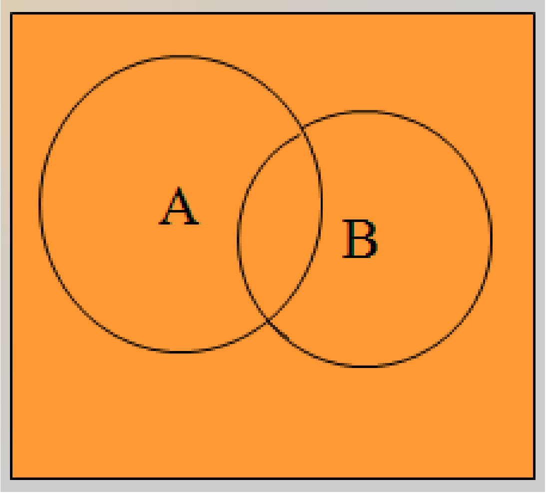
$$P(A \cup B) = 1$$



NonMutually Exclusive Events

- If an events A and B have intersection, we can say that

$$A \cap B \neq \emptyset$$



Properties of Probabilities…

(1) The probability of any outcome is between 0 and 1

$0 \leq P(O_i) \leq 1$ for each *i*, and

(2) The sum of the probabilities of all the outcomes equals 1

$P(O_1) + P(O_2) + \ldots + P(O_k) = 1$

$$\sum_{i=1}^{k} P(O_i) = 1$$

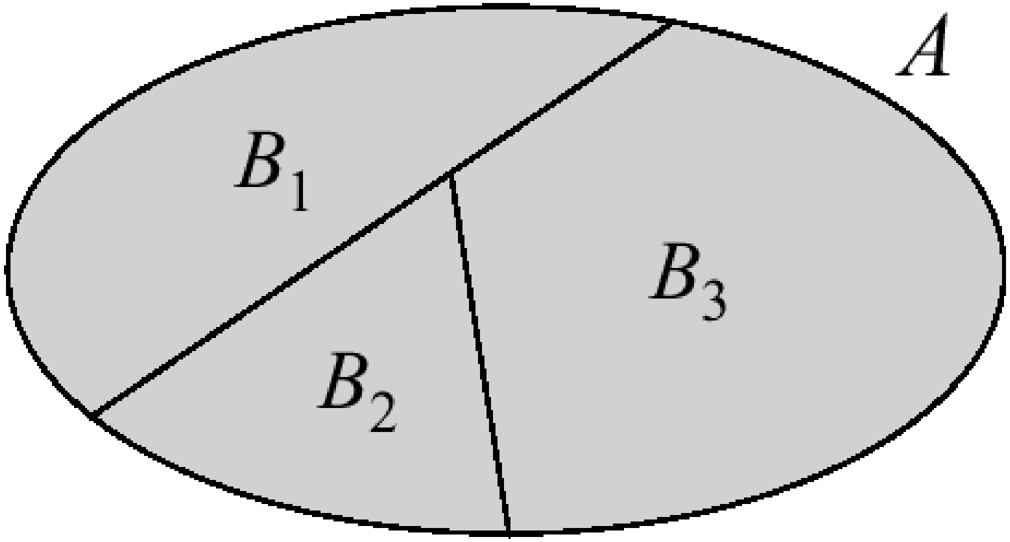
$P(O_i)$ represents the probability of outcome *i*

An event partition

- there are collection events {B1,B2,…} is the partition from an event A if satisfied :

  (i) $B_i \cap B_j = \varnothing$, for all $i \neq j$

  (ii) $\cup B_i = A$



For a discrete sample space, the *probability of an event E*, denoted as $P(E)$, equals the sum of the probabilities of the outcomes in E.

Properties of Probabilities…

$(i)\quad 0 \leq P(A) \leq 1$

$(ii)\quad P(\varnothing) = 0$

$(iii)\quad P(\Omega) = 1$

$(iv)\quad P(A^c) = 1 - P(A)$

$(v)\quad P(A \cup B) = P(A) + P(B) - \mathrm{P}(A \cap B)$

(vi) A and B are disjoint $\Rightarrow$ $P(A \cup B) = P(A) + P(B)$

(vii) $\{B_i\}$ is a partition of $A \Rightarrow$ $P(A) = \Sigma_i P(B_i)$

$(\mathrm{v}iii)$ $A \subset B \Rightarrow$ $P(A) \leq P(B)$

Events & Probabilities…

- **The *probability of an event* is the sum of the probabilities of the simple events that constitute the event.**

<u>Ex 13</u>

(assuming a fair die) S = {1, 2, 3, 4, 5, 6}

and

P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6

Then:

P(EVEN)

= P(2) + P(4) + P(6)

= 1/6 + 1/6 + 1/6 = 3/6 = 1/2

Examples14

- Experiment: Rolling ***dice***

  Sample Space: S = {2, 3, …, 12}

P(2) = 1/36

P(7) = 6/36

P(10) = 3/36

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12