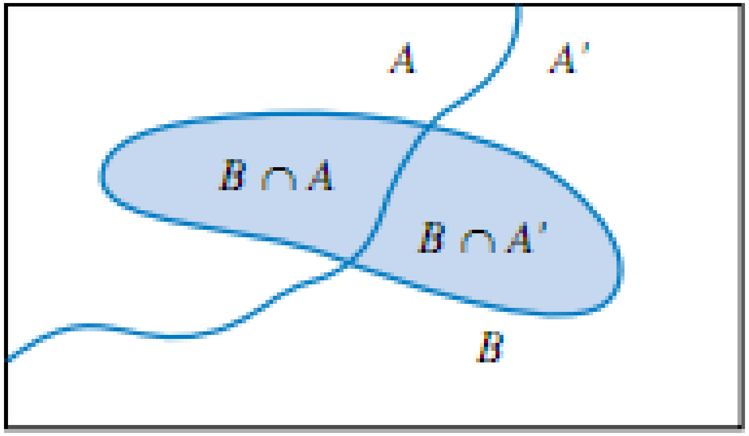
Probability

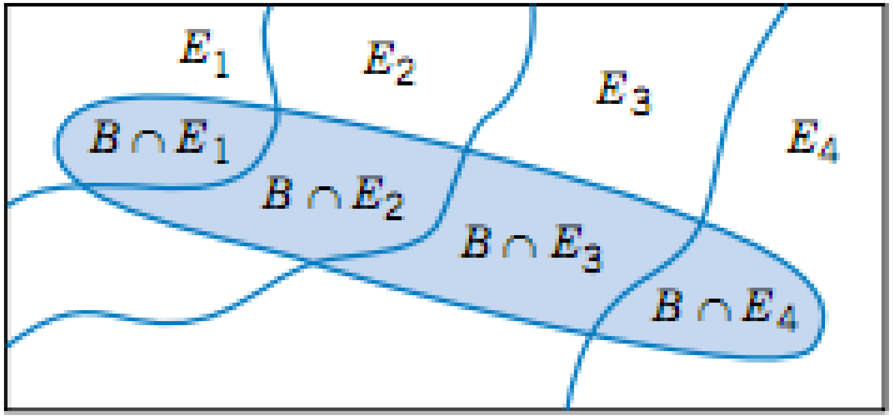
Part 3

- →Bayesian
- →Independency

Review… partitioning an event

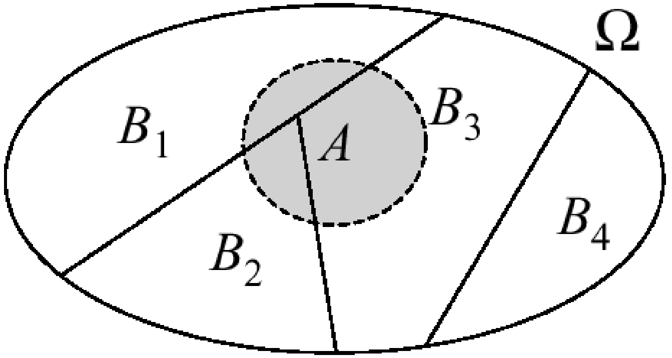


Partitioning an event into two mutually exclusive subsets.



$$B = (B \cap E_1) \cup (B \cap E_2) \cup (B \cap E_3) \cup (B \cap E_4)$$

Partitioning an event into several mutually exclusive subsets.



Total Probability Rule

- If $\{B_i\}$ is partition from *sample space* Ω
- then $\{A \cap B_i\}$ is partition events from event A, then based on probability properties , satiesfies :

$$P(A) = \sum_i P(A \cap B_i)$$

- Consider $P(B_i)>0$, for all of i

$$P(A) = \sum_i P(B_i) P(A \mid B_i)$$

Bayes Theorem

- If $\{B_i\}$ is partition from *sample space* Ω
- Assumed that $P(A)>0$ and $P(B_i)>0$, for all of i

$$P(B_i \mid A) = \frac{P(A \cap B_i)}{P(A)} = \frac{P(B_i)P(A|B_i)}{P(A)}$$

With total probability theorem,

$$P(B_i \mid A) = \frac{P(B_i)P(A|B_i)}{\sum_j P(B_j)P(A|B_j)}$$

- The equation above called Bayes Theorem
  - $P(B_i)$ called probability *a priori* from event B_i
  - $P(B_i | A)$ called probability *a posteriori* from event B_i (given event A)

Contoh

factory has three machines A, B and C to produce consecutive 60%, 30% and 10% of the total units produced many factories. The percentage of damage to products resulting from each successive machine is 2%, 3% and 4%. A unit selected at random and known to be faulty. Calculate the probability that the unit is derived from machine C.

Suppose R is the incidence of defective units, it would be calculated $P(C|R)$ is the probability that a unit produced by machine C with a known defective units

Dengan teorema Bayes, kejadian $P(A)$, $P(B)$ dan $P(C)$ adalah peluang (persentase produksi) dari masing-masing mesin; $P(R \mid A)$, $P(R \mid B)$ dan $P(R \mid C)$ adalah peluang (persentase kerusakan) dari masing-masing mesin.

$$
\begin{aligned}
P(C \mid R) &= \frac{P(C).P(R \mid C)}{P(A).P(R \mid A) + P(B).P(R \mid B) + P(C).P(R \mid C)} \\
&= \frac{(0,1)(0,04)}{(0,6)(0,02) + (0,3)(0,03) + (0,1)(0,04)} = \frac{4}{25}
\end{aligned}
$$

Exercise 1

2-70. Suppose that $P(A|B) = 0.4$ and $P(B) = 0.5$. Determine the following:

(a) $P(A \cap B)$

(b) $P(A' \cap B)$

2-71. Suppose that $P(A|B) = 0.2$, $P(A|B') = 0.3$, and $P(B) = 0.8$. What is $P(A)$?

2-72. The probability is 1% that an electrical connector that is kept dry fails during the warranty period of a portable computer. If the connector is ever wet, the probability of a failure during the warranty period is 5%. If 90% of the connectors are kept dry and 10% are wet, what proportion of connectors fail during the warranty period?

Definition : Independency

Two events are **independent** if any one of the following equivalent statements is true:

(1) $P(A|B) = P(A)$

(2) $P(B|A) = P(B)$

(3) $P(A \cap B) = P(A)P(B)$ (2-9)

The events $E_1, E_2, \ldots, E_n$ are independent if and only if for any subset of these events $E_{i_1}, E_{i_2}, \ldots, E_{i_k}$,

$$P(E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_k}) = P(E_{i_1}) \times P(E_{i_2}) \times \cdots \times P(E_{i_k}) \qquad (2\text{-}10)$$

Exercise 2

2-81. If $P(A|B) = 0.4$, $P(B) = 0.8$, and $P(A) = 0.5$, are the events A and B independent?

2-82. If $P(A|B) = 0.3$, $P(B) = 0.8$, and $P(A) = 0.3$, are the events B and the complement of A independent?

2-83. Disks of polycarbonate plastic from a supplier are analyzed for scratch and shock resistance. The results from 100 disks are summarized as follows:

		shock resistance	
		high	low
scratch	high	70	9
resistance	low	16	5

Let A denote the event that a disk has high shock resistance, and let B denote the event that a disk has high scratch resistance. Are events A and B independent?