Probability

Part 2

→ calculate probabilities

Union…

- The union of two events is denoted if the event that occurs when either or both event occurs. It is denoted as:

  A or B

- We can use this concept to answer questions like:

  Determine the probability that a fund outperforms the market ***or*** the manager graduated from a top-20 MBA program.

Example 1

Determine the probability that a fund outperforms (B_1) ***or*** the manager graduated from a top-20 MBA program (A_1).

A_1 and B_1 occurs, A_1 and B_2 occurs, or A_2 and B_1 occurs…

	B_1	B_2	$P(A_i)$
A_1	**.11**	**.29**	**.40**
A_2	**.06**	**.54**	**.60**
$P(B_j)$	**.17**	**.83**	**1.00**

$P(A_1 \text{ or } B_1) = .11 + .06 + .29 = \mathbf{.46}$

Example Union…

Determine the probability that a fund outperforms (B_1) ***or*** the manager graduated from a top-20 MBA program (A_1).

B_1

A_1

	B_1	B_2	$P(A_i)$
A_1	**.11**	**.29**	**.40**
A_2	**.06**	**.54**	**.60**
$P(B_j)$	**.17**	**.83**	**1.00**

$P(A_1 \text{ or } B_1) = .11 + .06 + .29 = \mathbf{.46}$

Probability Rules and Trees…

1. The Complement Rule
2. The Multiplication Rule
3. The Addition Rule

1. Complement Rule…

- The complement of an event A is the event that occurs when A does not occur.
- The ***complement rule*** gives us the probability of an event NOT occurring. That is:

$$P(A^C) = 1 – P(A)$$

Example 2

in the simple roll of a die, the probability of the number “1” being rolled is 1/6.

The probability that some number other than “1” will be rolled is 1 – 1/6 = 5/6.

2. Multiplication Rule…

The ***multiplication rule*** is used to calculate the ***joint probability*** of two events. It is based on the formula for conditional probability defined earlier:

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

If we multiply both sides of the equation by P(B) we have:

P(A and B) = P(A | B)•P(B)

Likewise, P(A and B) = P(B | A) • P(A)

If A and B are independent events, then

P(A and B) = P(A)•P(B)

3. Addition Rule…

Recall: the ***addition rule*** was introduced earlier to provide a way to compute the probability of event A ***or*** B ***or*** both A and B occurring; i.e. the union of A and B.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Addition Rule…

- $P(A_1) = .11 + .29 = .40$
- $P(B_1) = .11 + .06 = .17$
- By adding P(A) plus P(B) we add P(A and B) twice. To correct we subtract P(A and B) from P(A) + P(B)

B_1

A_1

	B_1	B_2	$P(A_i)$
A_1	.11	.29	.40
A_2	.06	**.54**	.60
$P(B_j)$	.17	.83	1.00

$P(A_1 \text{ or } B_1) = P(A) + P(B) - P(A \text{ and } B)$

$= .40 + .17 - .11$

$= \mathbf{.46}$

Find the probablity of :

$P(A_2 \text{ or } B_1)$

Addition Rule for Mutually Excusive Events

- If and A and B are mutually exclusive the occurrence of one event makes the other one impossible. This means that

  P(A and B) = 0

- The addition rule for mutually exclusive events is

  P(A or B) = P(A) + P(B)

- We often use this form when we add some joint probabilities calculated from a probability tree

Exercise 1

2-49. If $P(A) = 0.3$, $P(B) = 0.2$, and $P(A \cap B) = 0.1$, determine the following probabilities:

(a) $P(A')$ (b) $P(A \cup B)$

(c) $P(A' \cap B)$ (d) $P(A \cap B')$

(e) $P[(A \cup B)']$ (f) $P(A' \cup B)$

Conditional Probability

symbol $P(A \mid B)$ ← The probability of A given that we know B has happened.

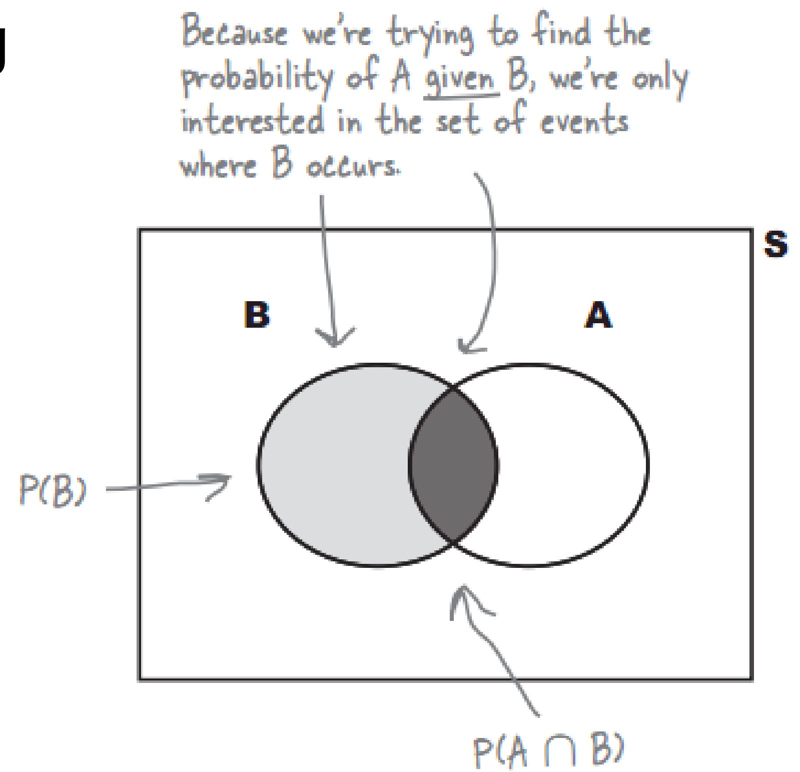
→ The probability of event A occurring given event B

→ What we're interested in

Is the number of outcome where both A and B occur, divided by all the B outcome

Or,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$



Example 3

- **DD Donuts are looking into the probabilites of their customers buying donuts and coffe. T. its know that P(Donuts)=3/4, P(Coffe|Donuts')=1/3 and P(Donuts∩Coffe)=9/20.**
- **Find P(Coffe|Donuts) !**

Exercise 2

2-57. Disks of polycarbonate plastic from a supplier are analyzed for scratch and shock resistance. The results from 100 disks are summarized as follows:

		shock resistance	
		high	low
scratch	high	70	9
resistance	low	16	5

Let A denote the event that a disk has high shock resistance, and let B denote the event that a disk has high scratch resistance. Determine the following probabilities:

(a) $P(A)$ (b) $P(B)$

(c) $P(A \mid B)$ (d) $P(B \mid A)$