Chapter 3

Hypothesis Testing

Curriculum Object

- Specified the problem based the form of hypothesis
- Student can arrange for hypothesis step
- Analyze a problem bassed for hypothesis

- Not everything you're told is absolutely certain
- Hypothesis tests → give a way using samples to test whether or not statistical claims are likely to be true

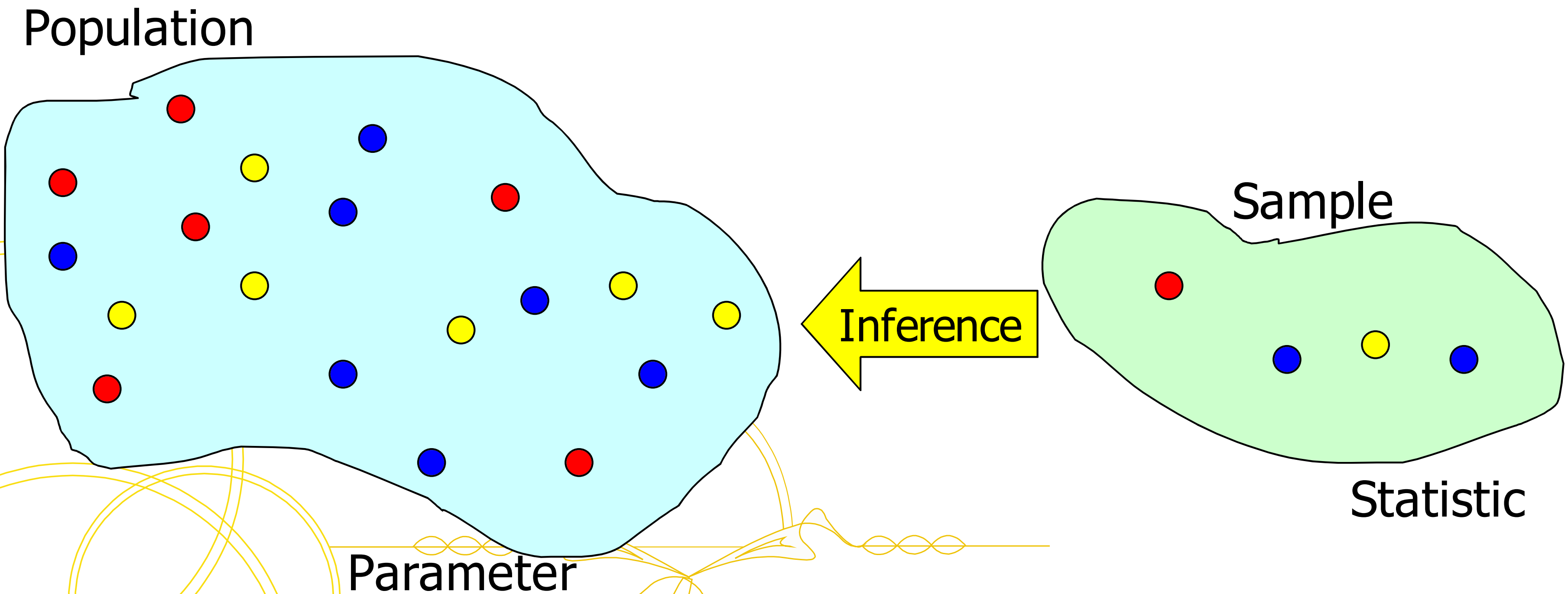
1. Examine the claim ; take the claim of the drug company

2. Examine the evidence ; see how much evidence we need to reject the drug company's claim and check this against the evidence we have

3. Make a decision ; depending on the evidence accept or reject the claims of the drug company

Introduction…

- **In addition to estimation, *hypothesis testing* is a procedure for making inferences about a population.**



RESume

- A hypothesis is a statement about a population parameter from one or more populations

- A hypothesis test is a procedure that
  - States the hypothesis to be tested
  - Uses sample information and formulates a decision rule
  - Based on the outcome of the decision rule the hypothesis is statistically validated or rejected

Steps for Hypothesis Testing

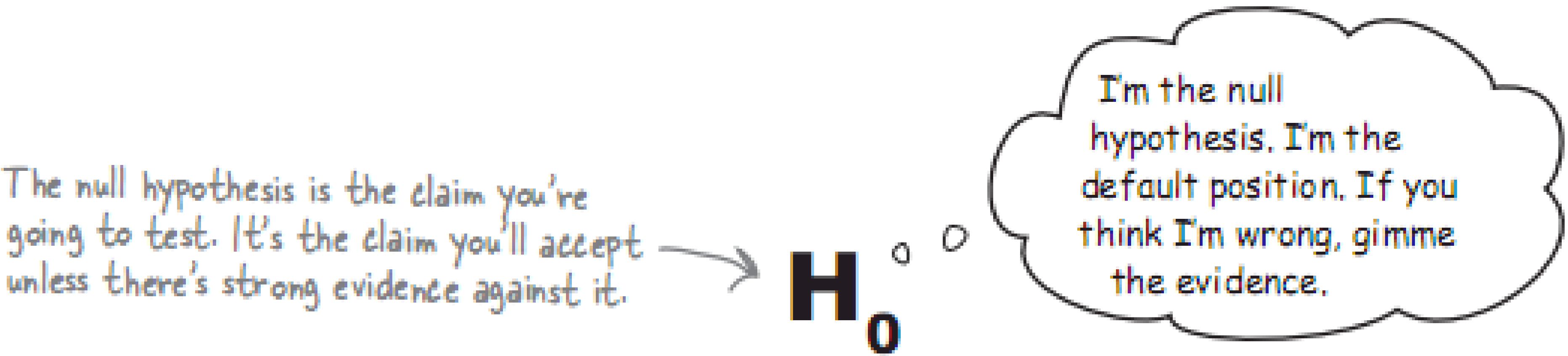
1. Decide on the hypothesis you're going to test
2. Choose your test statistic
3. Determine the critical region for your decision
4. Find the – p value of the test statistic
5. See whether the sample result is within the critical region
6. Make your decision

Step 1 : Decide on the hypothesis

Re :

Claim : cures 90% of patients

The claim that we're testing → null hypothesis, presented : H_0



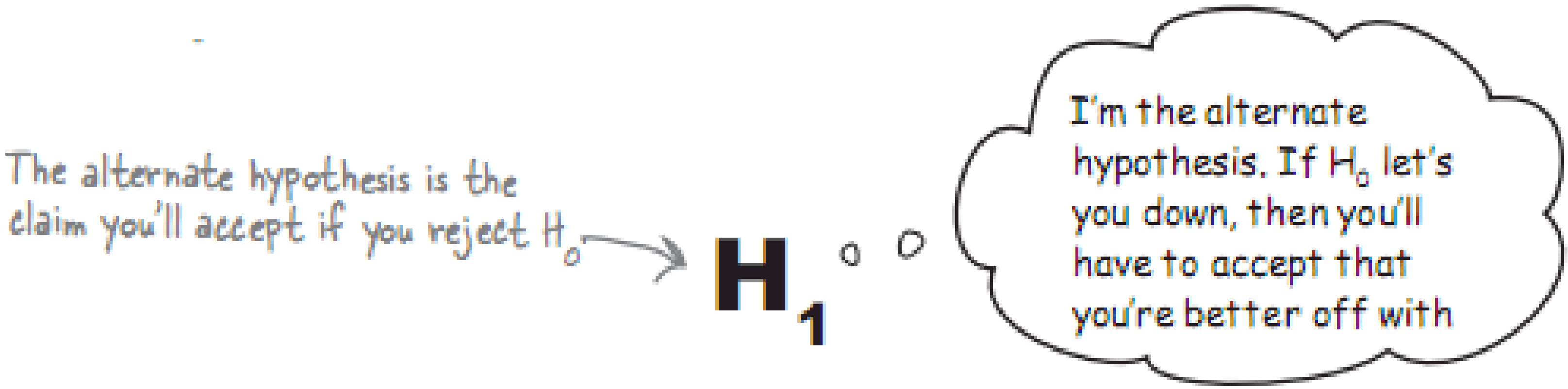
Null hypothesis (H_0)

- The hypothesis that there were no effects is called the NULL HYPOTHESIS.
- The null hypothesis states that in the general population there is no change, no difference, or no relationship.
- H_0 predicts that the independent variable (treatment) will have no effect on the dependent variable for the population.
- From the example SNORE CULL

H_0: p=90%=0.9

What if the claim is not true?

- The counterclaim to the null hypothesis is called alternative hypothesis, represented : $\mathbf{H_1}$



If the doctor believes that SNORE CULL cures less than 90% of people→ p<90%

H_1: p<90%=0.9

So…Alternative hypothesis (H_1)

- The alternative hypothesis (H_1) states that there is a change, a difference, or a relationship for the general population.
- H_1 is a statement of what a statistical hypothesis test is set up to establish.
- Form : H_1: $\mu_A \neq \mu_B$

- For example
  - H_1: the two drugs have different effects, on average.
  - H1: the new drug is better than the current drug, on average.

Concepts of Hypothesis Testing

- There are **two** hypotheses.

pronounced
H "nought"

- H_0: — *the 'null' hypothesis*
- H_1: — *the 'alternative' or 'research' hypothesis*

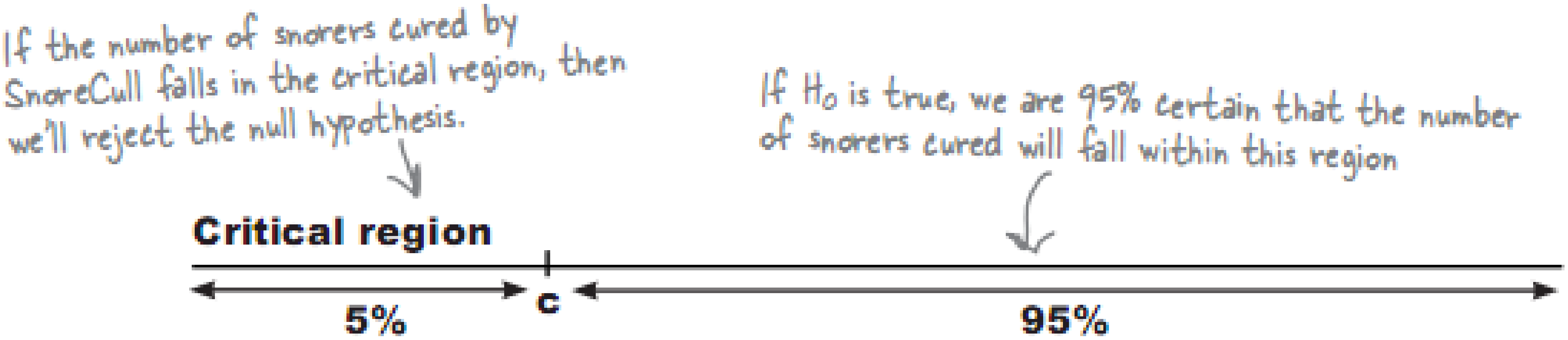
- **The null hypothesis (H_0) will always state that the *parameter <u>equals</u> the value* specified in the alternative hypothesis (H_1)**

Step 2: Choose your test statistic

Step 3: Determine the critical region

First decide level of significance

As an example suppose we want to test the claims of the drugs Company at a 5% level of significance.



Type I Error α

- A type I error is made when the researcher rejected the null hypothesis when it should not have been rejected.

- For example,
  - H_0: there is no difference between the two drugs on average.
- A type I error would occur if we concluded that the two drugs produced different effects when in fact there was no difference between them.
- P(type I error) = significance level = α
- What about with SNORE CULL…?

Type II Error

- A type II error is made when the null hypothesis is accepted when it should have been rejected.
- For example,
  - H_0 : there is no difference between the two drugs on average.
- A type II error would occur if it was concluded that the two drugs produced the same effect, i.e. there is no difference between the two drugs on average, when in fact they produced different ones.

- The probability of a type II error is generally unknown, represented :

  $P(\text{type II error}) = \beta$

RESUME

The following table gives a summary of possible results of any hypothesis test:

– **Decision**

– **Truth**

	Reject H_0	**Don't reject H_0**
H_0	Type I Error	Right decision
H_1	Right decision	Type II Error

$$\alpha = P(Type\ I\ \ Error) \quad \beta = P(Type\ II\ \ Error)$$

Goal: Keep α, β reasonably small

Example

For example, suppose that we are interested in the burning rate of a solid propellant used to power aircrew escape systems. Now burning rate is a random variable that can be described by a probability distribution. Suppose that our interest focuses on the mean burning rate (a parameter of this distribution). Specifically, we are interested in deciding whether or not the mean burning rate is 50 centimeters per second. We may express this formally as

So, We have 3 kinds of hypothesis :

H_0: $\mu = 50$ centimeters per second

H_1: $\mu \neq 50$ centimeters per second

H_0: $\mu = 50$ centimeters per second

H_1: $\mu < 50$ centimeters per second

H_0: $\mu = 50$ centimeters per second

H_1: $\mu > 50$ centimeters per second

Steps in applying Hypothesis testing :

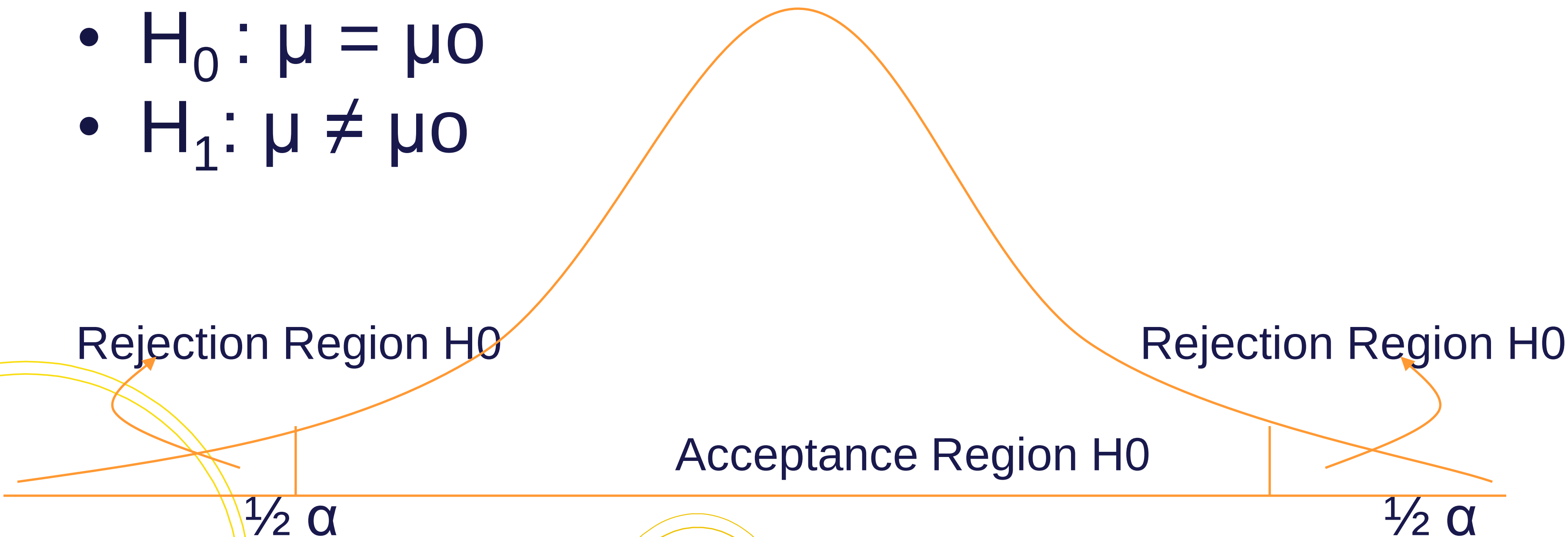
i. **State The Hypothesis**

$$a.\ H_0 : \mu = \mu_0$$
$$H_1 : \mu \neq \mu_0$$
$$b.\ H_0 : \mu = \mu_0$$
$$H_1 : \mu > \mu_0$$
$$c.\ H_0 : \mu = \mu_0$$
$$H_1 : \mu < \mu_0$$

ii. **Choose a significance level**

Two Tailed Test

- $H_0 : \mu = \mu o$
- $H_1 : \mu \neq \mu o$



iii. H_0 accepted if: $-z_{1/2\alpha} < z < z_{1/2\,\alpha}$

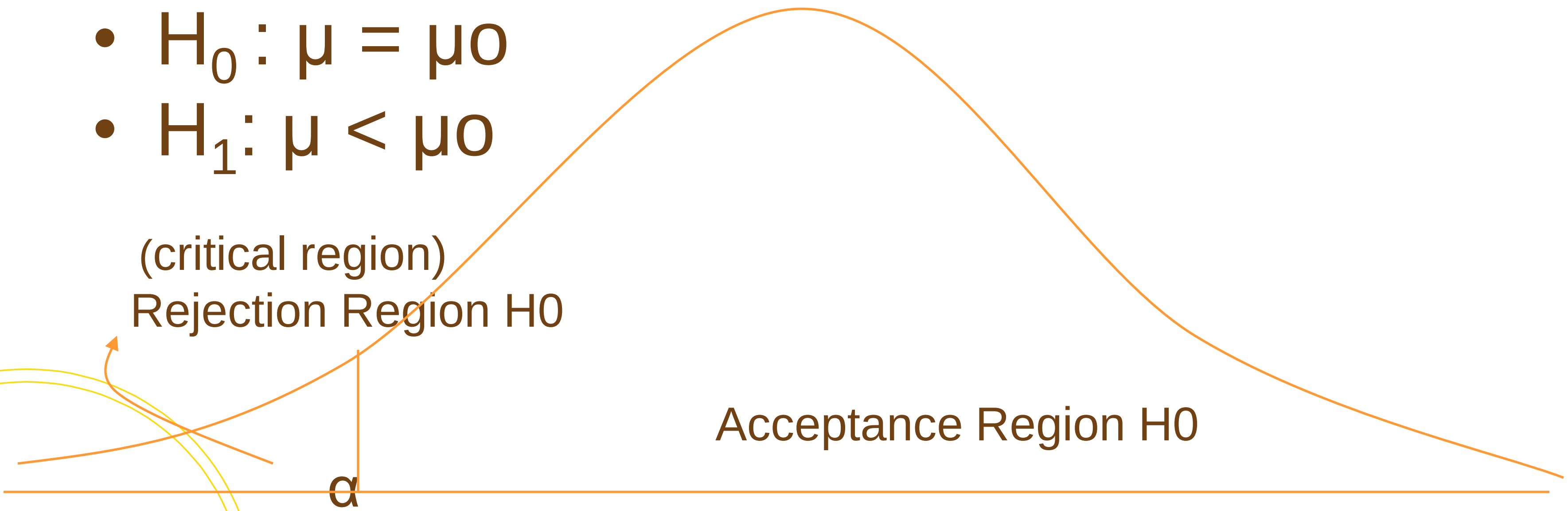
One Tailed Test (right)

- $H_0 : \mu = \mu o$
- $H_1 : \mu > \mu o$

iii. H_0 accepted if : $z \leq z_{\alpha}$

One tailed (left)

- $H_0 : \mu = \mu o$
- $H_1 : \mu < \mu o$



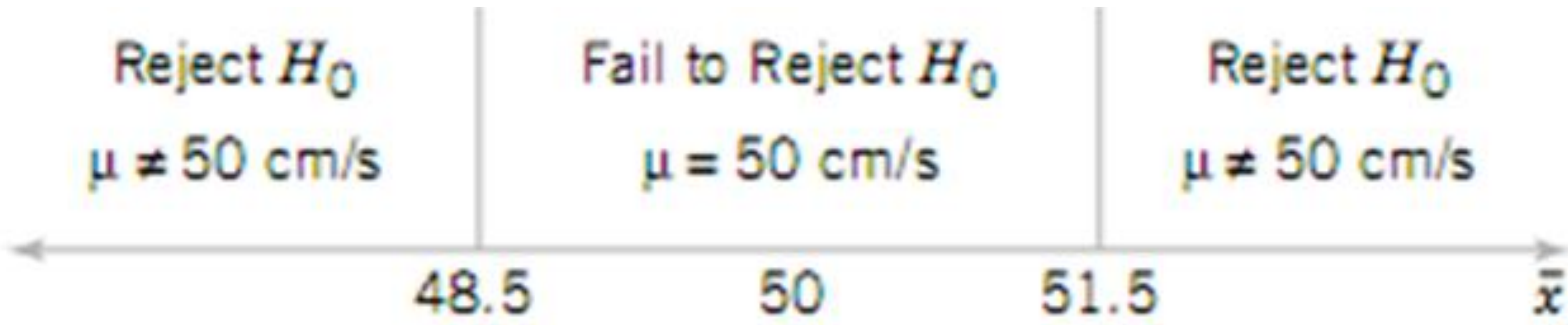
iii. H_0 accepted if: $z \geq -z_\alpha$

iv. Calculation :

$$Z = \frac{\bar{X} - \theta_0}{\sigma / \sqrt{n}}$$

$$Z = \frac{\bar{X} - \theta_0}{s / \sqrt{n}}, \sigma \text{ unknown}$$

Suppose : $48.5 \le \bar{x} \le 51.5$

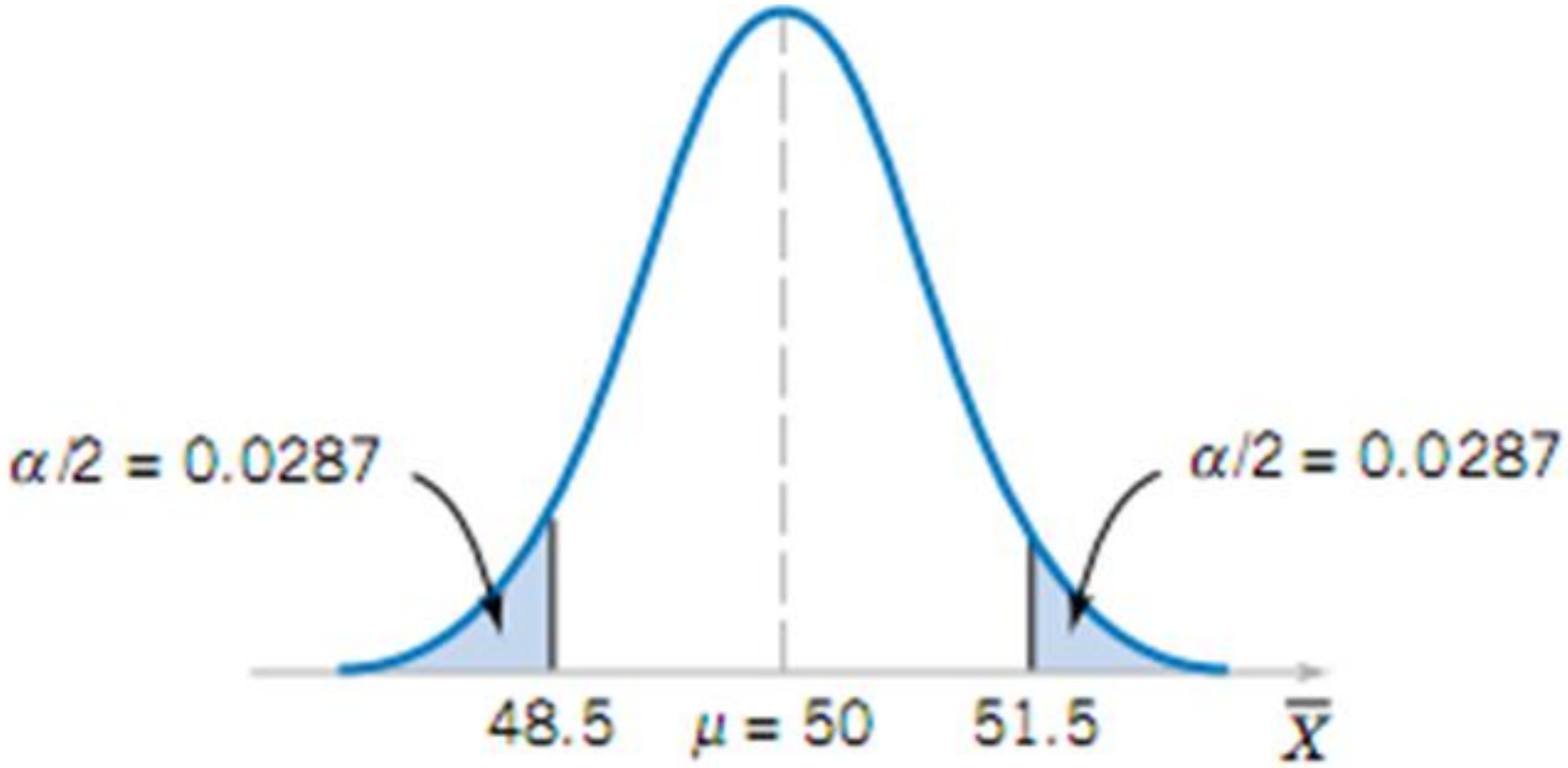


$$\alpha = P(\text{Type I Error})$$
$$= P(\text{Reject } H_0, H_0 \text{ is true})$$
$$= P(\overline{X} < 48.5, \quad \text{with } \mu = 50) + P(\overline{X} > 51.5, \quad \text{with } \mu = 50)$$

suppose $n = 10, \sigma = 2.5,$

$$z_1 = \frac{48.5 - 50}{2.5/\sqrt{10}} = -1.90 \qquad z_2 = \frac{51.5 - 50}{2.5/\sqrt{10}} = 1.90$$

$$\alpha = P(Z < -1.90) + P(Z > 1.90) = 0.028717 + 0.028717 = 0.057434$$



Implies that 5.76% of all random samples would lead to rejection of the Ho when Ho is true

β

$$\beta = P\left(48.5 \le \overline{X} \le 51.5, \quad \mu = 52\right)$$

$$z_1 = \frac{48.5 - 52}{2.5/\sqrt{10}} = -4.43$$

$$z_2 = \frac{51.5 - 52}{2.5/\sqrt{10}} = -0.63$$

$$\beta = P\left(48.5 \le \overline{X} \le 51.5, \quad \mu = 52\right)$$

$$= P(z_2) - P(z_1)$$

$$= 0.2643 - 0.0000$$

$$= 0.2643$$

at previous example, we have β=0.2643 the we have **the power of this test** is 1- β=0.7357 when μ=52

→At previous ex :

the sensitivity of the test for detecting the difference between a mean burning rate of 50 cm/s and 52 cm/s is 0.7357

→If the true mean is really 52cm/s this test will correctly reject H0: μ=50 and detect this difference 73.57% of the time

Exercise

9-1. In each of the following situations, state whether it is a correctly stated hypothesis testing problem and why.

(a) H_0: $\mu = 25$, H_1: $\mu \neq 25$
(b) H_0: $\sigma > 10$, H_1: $\sigma = 10$
(c) H_0: $\bar{x} = 50$, H_1: $\bar{x} \neq 50$
(d) H_0: $p = 0.1$, H_1: $p = 0.5$
(e) H_0: $s = 30$, H_1: $s > 30$

9-2. A textile fiber manufacturer is investigating a new drapery yarn, which the company claims has a mean thread elongation of 12 kilograms with a standard deviation of 0.5 kilograms. The company wishes to test the hypothesis H_0: $\mu = 12$ against H_1: $\mu < 12$, using a random sample of four specimens.

(a) What is the type I error probability if the critical region is defined as $\bar{x} < 11.5$ kilograms?
(b) Find β for the case where the true mean elongation is 11.25 kilograms.