Test On the Mean of a Normal Distribution, Variance Unknown

Hypothesis Tests on The Mean

Null hypothesis: $H_0\text{: } \mu = \mu_0$

Test statistic: $T_0 = \dfrac{\overline{X} - \mu_0}{S/\sqrt{n}}$

Alternative hypothesis	Rejection criteria
$H_1\text{: } \mu \neq \mu_0$	$t_0 > t_{\alpha/2,n-1}$ or $t_0 < -t_{\alpha/2,n-1}$
$H_1\text{: } \mu > \mu_0$	$t_0 > t_{\alpha,n-1}$
$H_1\text{: } \mu < \mu_0$	$t_0 < -t_{\alpha,n-1}$

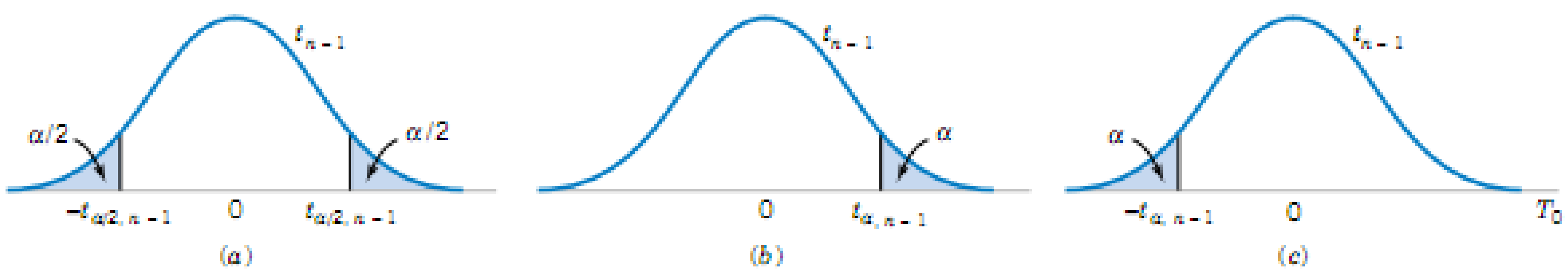

Figure 9-8 The reference distribution for $H_0\text{: } \mu = \mu_0$ with critical region for (a) $H_1\text{: } \mu \neq \mu_0$, (b) $H_1\text{: } \mu > \mu_0$, and (c) $H_1\text{: } \mu < \mu_0$.

I. Test on the Mean of a normal Distribution, variance known

- Suppose that we wish to test the hypothesis :

$$H_0: \mu = \mu_0$$
$$H_1: \mu \neq \mu_0$$

- Test Statistics :

$$Z_0 = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}$$

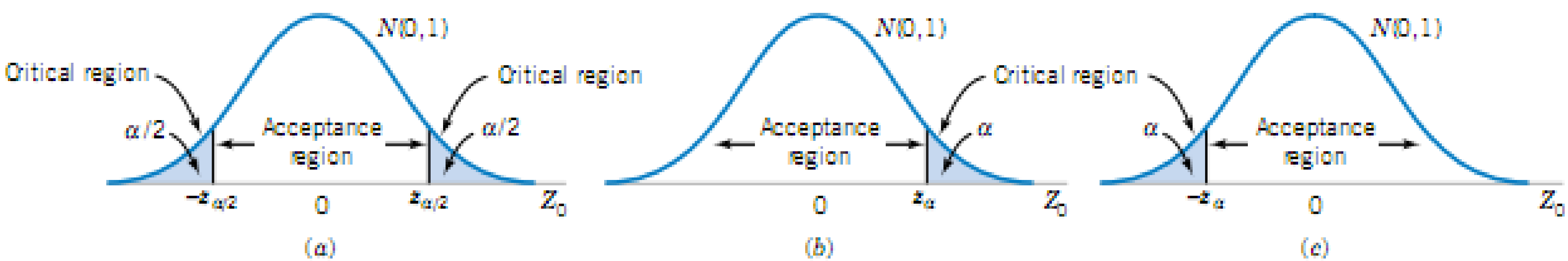

Figure 9-6 The distribution of Z_0 when H_0: $\mu = \mu_0$ is true, with critical region for (a) the two-sided alternative $H_1: \mu \neq \mu_0$, (b) the one-sided alternative $H_1: \mu > \mu_0$, and (c) the one-sided alternative $H_1: \mu < \mu_0$.

Example

Aircrew escape systems are powered by a solid propellant. The burning rate of this propellant is an important product characteristic. Specifications require that the mean burning rate must be 50 centimeters per second. We know that the standard deviation of burning rate is $\sigma = 2$ centimeters per second. The experimenter decides to specify a type I error probability or significance level of $\alpha = 0.05$ and selects a random sample of $n = 25$ and obtains a sample average burning rate of $\bar{x} = 51.3$ centimeters per second. What conclusions should be drawn?

1. The parameter of interest is μ, the mean burning rate.
2. H_0: $\mu = 50$ centimeters per second
3. H_1: $\mu \neq 50$ centimeters per second
4. $\alpha = 0.05$
5. The test statistic is

$$z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

6. Reject H_0 if $z_0 > 1.96$ or if $z_0 < -1.96$. Note that this results from step 4, where we specified $\alpha = 0.05$, and so the boundaries of the critical region are at $z_{0.025} = 1.96$ and $-z_{0.025} = -1.96$.
7. Computations: Since $\bar{x} = 51.3$ and $\sigma = 2$,

$$z_0 = \frac{51.3 - 50}{2/\sqrt{25}} = 3.25$$

8. Conclusion: Since $z_0 = 3.25 > 1.96$, we reject H_0: $\mu = 50$ at the 0.05 level of significance. Stated more completely, we conclude that the mean burning rate differs from 50 centimeters per second, based on a sample of 25 measurements. In fact, there is strong evidence that the mean burning rate exceeds 50 centimeters per second.