Chapter 4

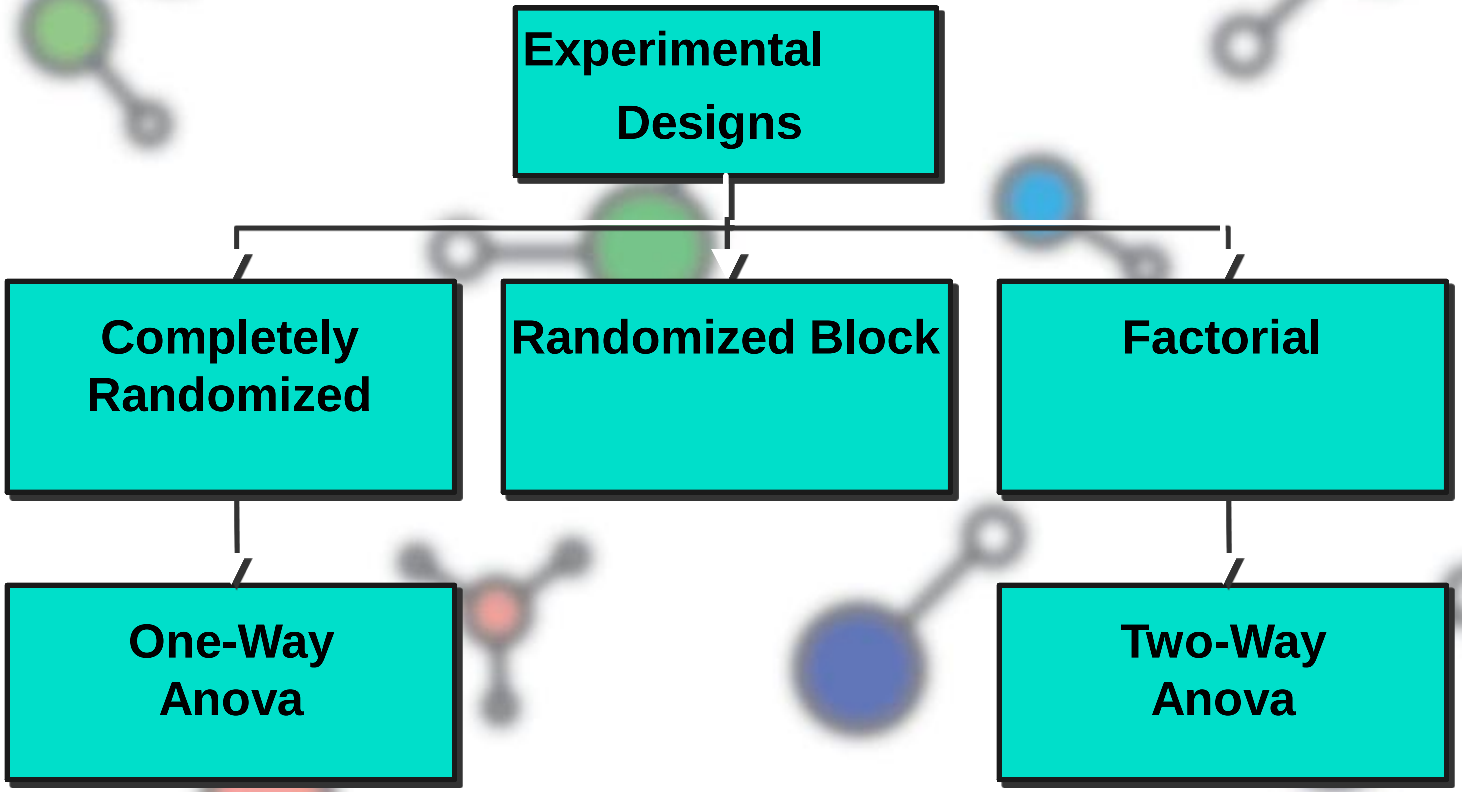
Analysis of Variance

Experiment

1. Investigator Controls (Or Observes) One or More Independent Variables
   - Called Treatment Variables or Factors
   - Contain Two or More Levels (Subcategories)
   - Treatments are combinations of factor-levels for the different factors
2. Observes Effect on Dependent Variable
   - Response to Levels of Independent Variable
3. Experimental Design: Plan Used to Test Hypotheses

Examples of Experiments

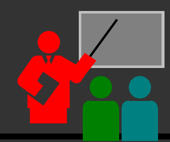
1. Thirty Stores Are Randomly Assigned 1 of 4 (Levels) Store Displays (Independent Variable) to See the Effect on Sales (Dependent Variable).
2. Two Hundred Consumers Are Randomly Assigned 1 of 3 (Levels) Brands of Juice (Independent Variable) to Study Reaction (Dependent Variable).


Experimental Designs
Completely Randomized
Randomized Block
Factorial
One-Way Anova
Two-Way Anova

Completely Randomized Design

1. Experimental Units (Subjects) Are Assigned Randomly to Treatments
   – Subjects are Assumed Homogeneous
2. One Factor or Independent Variable
   – 2 or More Treatment Levels or Classifications
3. Analyzed by One-Way ANOVA

Randomized Design Example

	Factor (Training Method)		
Factor levels (Treatments)	**Level 1** 	**Level 2** 	**Level 3** 
Experimental units	😐 😐 😐	😐 😐 😐	😐 😐 😐
Dependent variable (Response)	21 hrs.	17 hrs.	31 hrs.
	27 hrs.	25 hrs.	28 hrs.
	29 hrs.	20 hrs.	22 hrs.

One-Way ANOVA F-Test Assumptions

1. Randomness & Independence of Errors
   - Independent Random Samples are Drawn for each condition
2. Normality
   - Populations (for each condition) are Normally Distributed
3. Homogeneity of Variance
   - Populations (for each condition) have Equal Variances

Single Factor Analysis of Variance

one- way or *single factor analysis of variance* , **do you know why?**

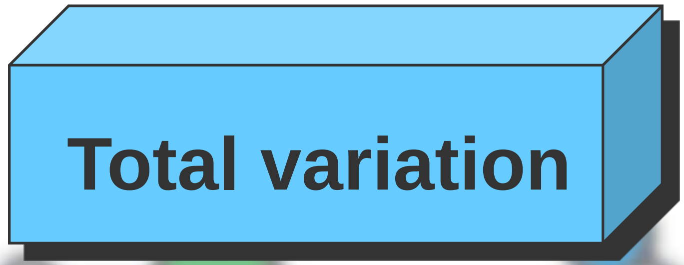
→ *completely randomized design*

- Model :

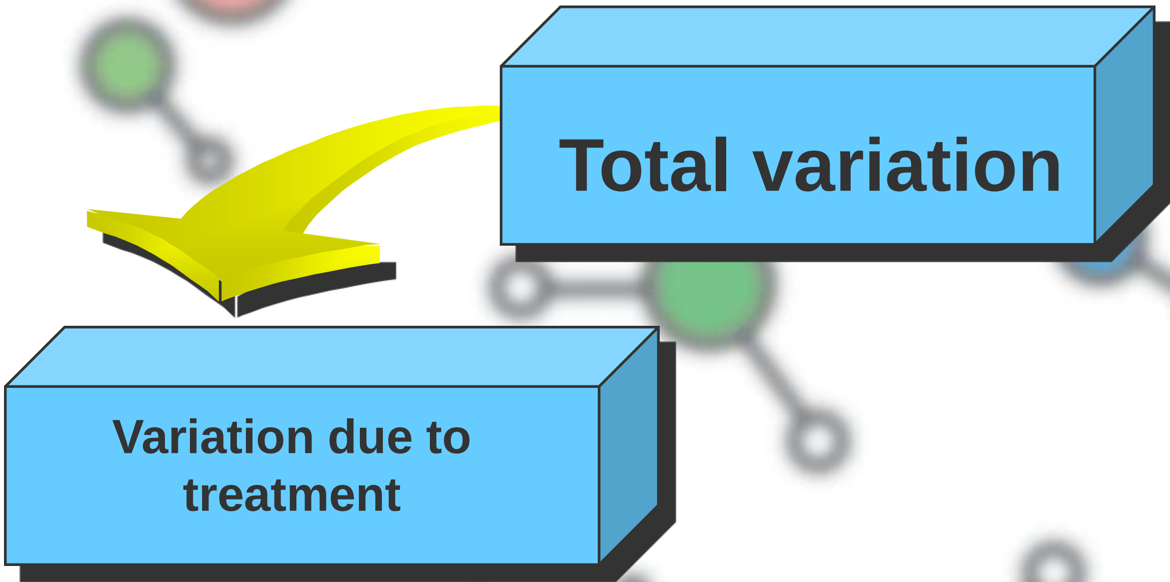
$$y_{ij} = \mu + \tau_i + \varepsilon_{ij} \begin{cases} i = 1,..,a \\ j = 1,..,n \end{cases}$$

One-Way ANOVA
Partitions Total Variation

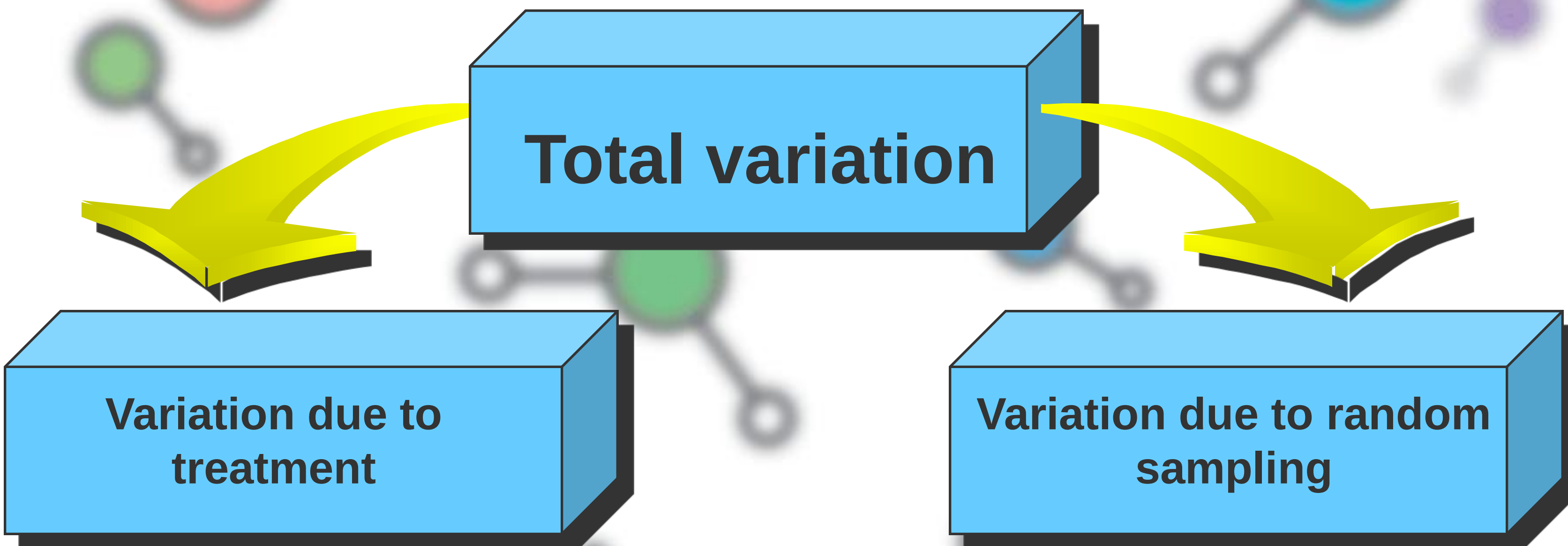
One-Way ANOVA Partitions Total Variation



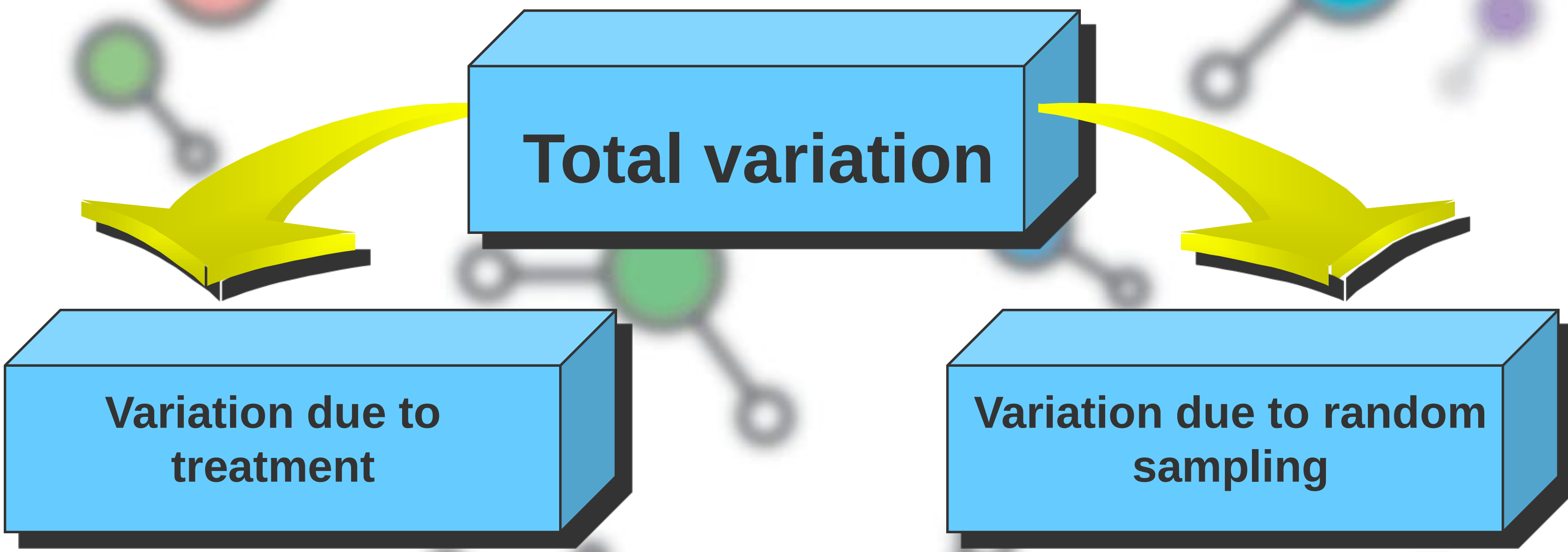
One-Way ANOVA Partitions Total Variation



One-Way ANOVA Partitions Total Variation

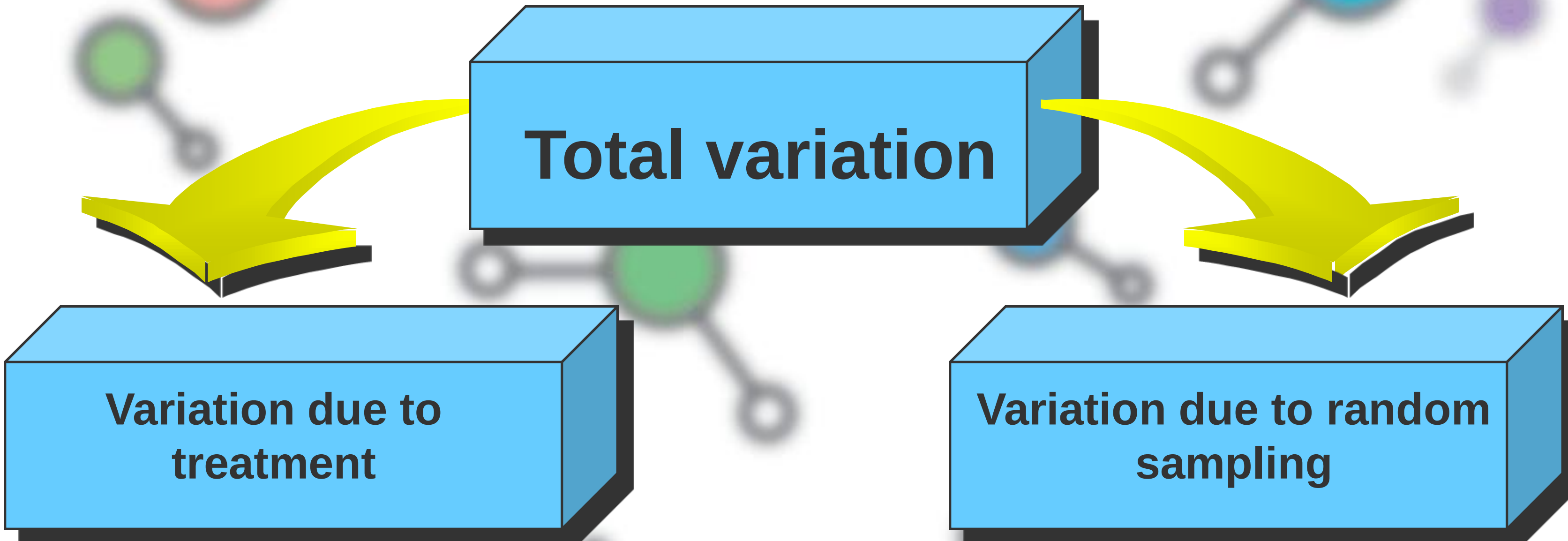


One-Way ANOVA Partitions Total Variation



Sum of Squares Among
Sum of Squares Between
Sum of Squares Treatment
Among Groups Variation

One-Way ANOVA Partitions Total Variation



- Sum of Squares Among
- Sum of Squares Between
- **Sum of Squares Treatment (SST)**
- Among Groups Variation

- Sum of Squares Within
- **Sum of Squares Error (SSE)**
- Within Groups Variation

$$y_{ij} - \bar{y}_{\bullet\bullet} = \bar{y}_{i\bullet} - \bar{y}_{\bullet\bullet} + y_{ij} - \bar{y}_{i\bullet}$$

$$\sum_{i=1}^{a}\sum_{j=1}^{n}\left\{y_{ij} - \bar{y}_{\bullet\bullet}\right\}^2 = \sum_{i=1}^{a}\sum_{j=1}^{n}\left\{\bar{y}_{i\bullet} - \bar{y}_{\bullet\bullet} + y_{ij} - \bar{y}_{i\bullet}\right\}^2$$

$$\underbrace{\sum_{i=1}^{a}\sum_{j=1}^{n}\left\{y_{ij} - \bar{y}_{\bullet\bullet}\right\}^2}_{SS_{\mathrm{T}}} = \underbrace{\sum_{i=1}^{a}\sum_{j=1}^{n}\left\{\bar{y}_{i\bullet} - \bar{y}_{\bullet\bullet}\right\}^2}_{SS_{\mathrm{P}}} + \underbrace{\sum_{i=1}^{a}\sum_{j=1}^{n}\left\{y_{ij} - \bar{y}_{i\bullet}\right\}^2}_{SS_{\mathrm{S}}}$$

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{n}\left\{y_{ij} - \bar{y}_{\bullet\bullet}\right\}^2 = \sum_{i}^{a}\sum_{j}^{n} y_{ij}^2 - \frac{y_{\bullet\bullet}^2}{N}$$

$$SS_P = \sum_{i=1}^{a}\sum_{j=1}^{n}\left\{\bar{y}_{i\bullet} - \bar{y}_{\bullet\bullet}\right\}^2 = \sum_{i=1}^{a}\frac{y_{i\bullet}^2}{n} - \frac{y_{\bullet\bullet}^2}{N}$$

$$SS_E = \sum_{i=1}^{a}\sum_{j=1}^{n}\left\{y_{ij} - \bar{y}_{i\bullet}\right\}^2 = \mathrm{JK_T} - \mathrm{JK_P}$$

Perlakuan ke-	Observasi				Total	Rata-rata
1	y_{11}	y_{12}	$\cdots$	y_{1n}	$y_{1\bullet}$	$\bar{y}_{1\bullet}$
2	y_{21}	y_{22}	$\cdots$	y_{2n}	$y_{2\bullet}$	$\bar{y}_{2\bullet}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	
a	y_{a1}	y_{a2}	$\cdots$	y_{an}	$y_{a\bullet}$	$\bar{y}_{a\bullet}$
Jumlah					$y_{\bullet\bullet}$	$\bar{y}_{\bullet\bullet}$

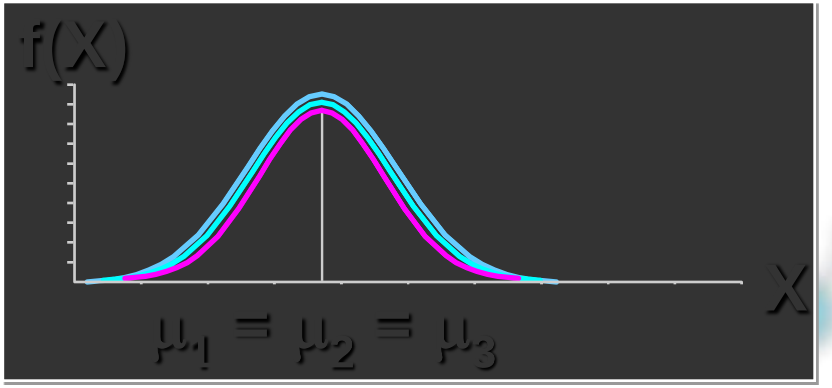
Keterangan :

$$y_{i\bullet} = \sum_{j=1}^{n} y_{ij}, \quad y_{\bullet\bullet} = \sum_{i=1}^{a}\sum_{j=1}^{n} y_{ij}, \quad \bar{y}_{i\bullet} = \frac{y_{i\bullet}}{n}, \quad \bar{y}_{\bullet\bullet} = \frac{y_{\bullet\bullet}}{N}, \; i = 1, \ldots, a$$

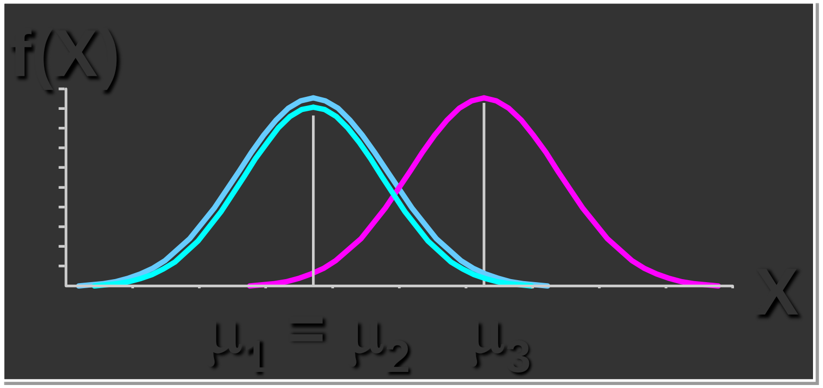
$$N = an$$

One-Way ANOVA F-Test Hypothesis

1. H_0: $\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_p$
   - All Population Means are Equal
   - No Treatment Effect



H_a: Not All μ_j Are Equal
   - At Least 1 Pop. Mean is Different
   - Treatment Effect
   - NOT $\mu_1 \neq \mu_2 \neq \ldots \neq \mu_p$



ii. Take any α

iii. Tabel ANOVA 1 Jalan

Source of Variance	SS	df	MS	Fo
Treatment	SST	a-1	MSP=SST/(a-1)	Fp=MSP/MSE
Error	SSE	a(n-1)	MSE=SSE/a(n-1)	
Total	SST	an-1		

iv. Critical Region

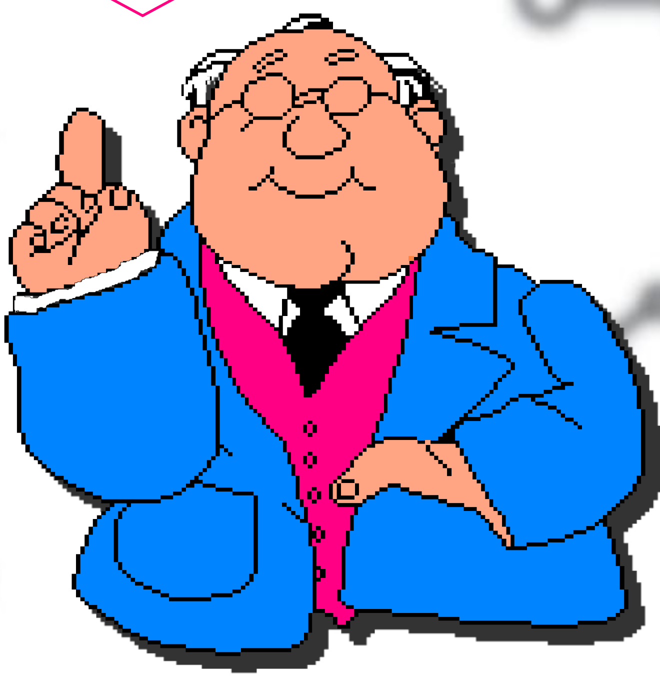
$$\text{Reject } H_0 \text{ If } Fo > F_{df(treatment),db(error)}$$

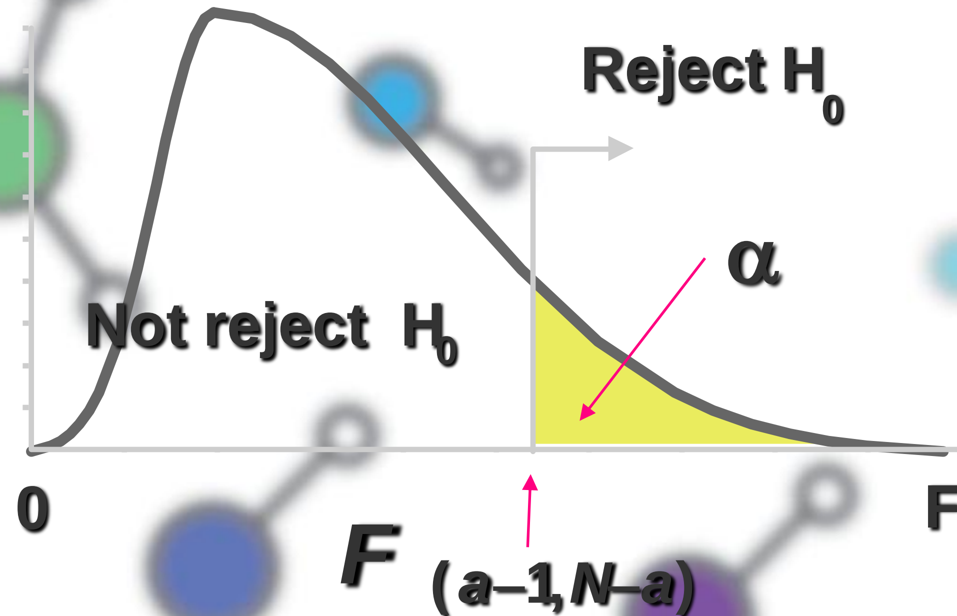
or

$$\text{Reject } H_0 \text{ If } Fo > F_{(a-1),(a(n-1))}$$

One-Way ANOVA F-Test Critical Value

If means are equal, $F = MST / MSE \approx 1$. Only reject large F!





Always One-Tail!

One-Way ANOVA F-Test Example

Mach1	Mach2	Mach3
25.40	23.40	20.00
26.31	21.80	22.20
24.10	23.50	19.75
23.74	22.75	20.60
25.10	21.60	20.40

As production manager, you want to see if 3 filling machines have different mean filling times. You assign 15 similarly trained & experienced workers, 5 per machine, to the machines. At the .05 level, is there a difference in mean filling times?

One-Way ANOVA F-Test Solution

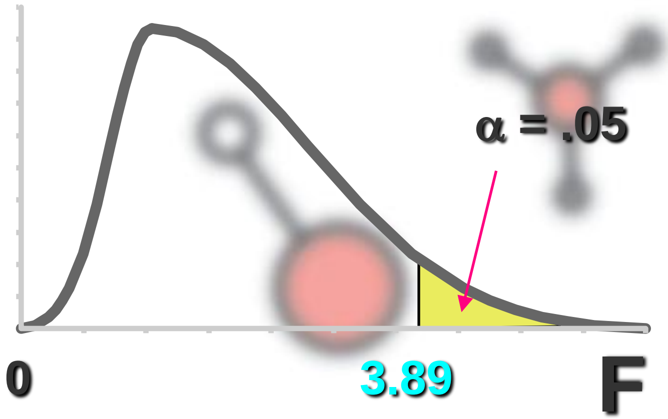
H0: $\mu_1 = \mu_2 = \mu_3$

Ha: Not All Equal

α = .05

$\nu_1 = 2$ $\nu_2 = 12$

Critical Value(s):



Test Statistic:

$$F = \frac{MST}{MSE} = \frac{23.5820}{.9211} = 25.6$$

Decision:

Reject at α = .05

Conclusion:

There Is Evidence Pop. Means Are Different

Summary Table Solution

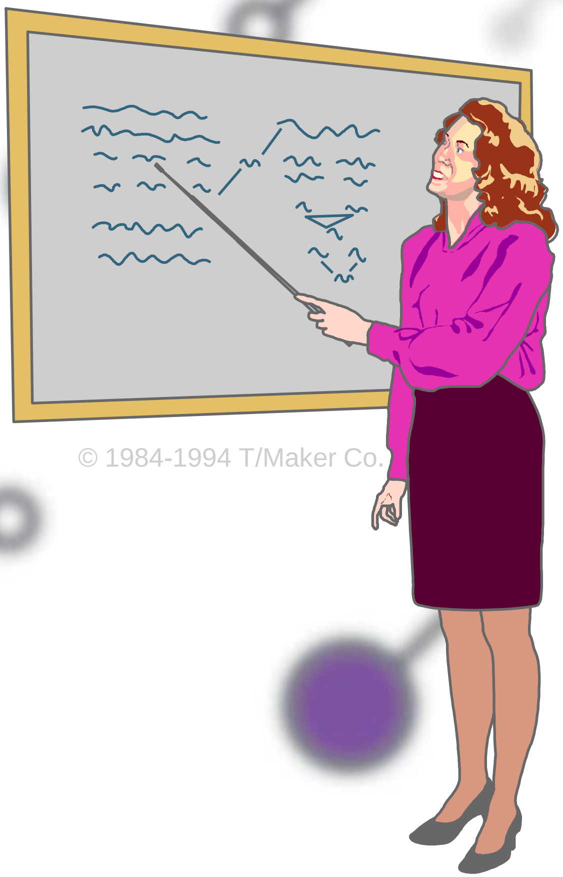
Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square (Variance)	F
Treatment (Machines)	3 - 1 = 2	47.1640	23.5820	25.60
Error	15 - 3 = 12	11.0532	.9211	
Total	15 - 1 = 14	58.2172		

One-Way ANOVA F-Test Thinking Challenge

You're a trainer for Microsoft Corp. Is there a difference in **mean** learning times of 12 people using 4 different training methods (α =.05)?

M1	M2	M3	M4
10	11	13	18
9	16	8	23
5	9	9	25

Use the following table.



Summary Table (Partially Completed)

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F
Treatment (Methods)		348		
Error		80		
Total				

One-Way ANOVA F-Test Solution*

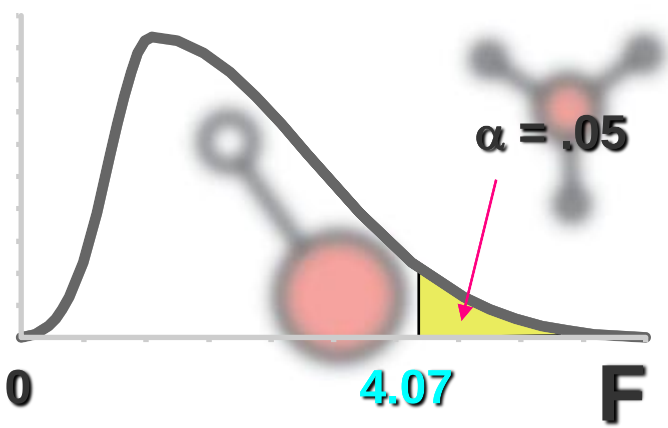
H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$

H_a: Not All Equal

$\alpha = .05$

$\nu_1 = 3$ $\nu_2 = 8$

Critical Value(s):



Test Statistic:

$$F = \frac{MST}{MSE} = \frac{116}{10} = 11.6$$

Decision:

Reject at $\alpha = .05$

Conclusion:

There Is Evidence Pop. Means Are Different