# Chapter 1

## Point Estimation part 1
## : Methods of Estimation

# Definition

A statistic, $T = l(X_1, X_2, \ldots, X_n)$ that is used to estimate the value of $\tau(\theta)$ is called an estimator of $\tau(\theta)$, and an observed value of the statistic, $t = l(x_1, x_2, \ldots, x_n)$ is called an estimate of $\tau(\theta)$

**Mostly methods on Point estimation** :
→**Moments**
→**Maximum Likelihood**

# 1. Method of Moment

## Definition 9.2.1 pg 291

If $X_1, X_2, \ldots, X_n$ is a random sample from $f\left(x;\theta_1,\ldots,\theta_k\right)$

the first $k$ sample moments are given $M_j^{'} = \dfrac{\sum_{i=1}^{n} X_i^j}{n}, \; j = 1,..,k$

# Example 9.2.1

A r.V sample from a distribution with two unknown parameters the mean $\mu$ and the variance $\sigma^2$ then find $\hat{\mu}$ and $\hat{\sigma}^2$

# Example 9.2.3

**A r.V sample from an exponential distribution,** $X_i \sim EXP(\theta)$ **then estimate the probability** $p(\theta) = P(X \geq 1)$

# 2. Method of Maximum Likelihood

## Definition 9.2.2

The joint density function of $n$ random variable $X_1, X_2, \ldots, X_n$ evaluated at $x_1, x_2, \ldots, x_n$ say $f(x_1, x_2, \ldots, x_n; \theta)$ is refered to as Likelihood function.

For fixed $x_1, x_2, \ldots, x_n$ the likelihood function is a function of $\theta$ and often denoted by $L(\theta)$

If $X_1, X_2, \ldots, X_n$ represent a random sample from $f(x;\theta)$, then

$L(\theta) = f(x_1;\theta) \cdots f(x_n;\theta)$

# Definition 9.2.3

Let $L(\theta) = f(x_1, \ldots, x_n; \theta), \theta \in \Omega$, be the joint pdf of $X_1, X_2, \ldots, X_n$.

For given set of observations, $(x_1, x_2, \ldots, x_n)$ a value $\hat{\theta}$ in $\Omega$ at which $L(\theta)$ is a maximum likelihood estimate (MLE) of $\theta$ is a value of $\theta$ that satisfies:

$$f\left(x_1, \ldots, x_n; \hat{\theta}\right) = \max_{\theta \in \Omega} f(x_1, \ldots, x_n; \theta)$$

# Th. Invariance Property

If $\hat{\theta}$ is MLE of $\theta$ and if $u(\theta)$ is a function of $\theta$ then $u\left(\hat{\theta}\right)$ is an MLE of $u(\theta)$

If $\hat{\theta} = \left(\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_k\right)$ denotes the MLE of $\theta = (\theta_1, \theta_2, \ldots, \theta_k)$ then the MLE of $\tau = (\tau_1(\theta), \tau_2(\theta), \ldots, \tau_r(\theta))$ is

$$\hat{\tau} = (\hat{\tau}_1, \hat{\tau}_2, \ldots, \hat{\tau}_r) = (\hat{\tau}_1(\theta), \hat{\tau}_2(\theta), \ldots, \hat{\tau}_r(\theta)) \text{ for } \quad 1 \le r \le k$$

# Example

1. If $X_i \sim POI(\theta)$ then find $\hat{\theta}$
2. If $X_i \sim EXP(\theta)$ then find $\hat{\theta}$
3. If $X_i \sim N(\mu, \sigma^2)$ then find $\hat{\mu}, \hat{\sigma}^2$