Chapter 2

SUFFICIENCY & COMPLETENESS

SUFFICIENCY

Idea → the reduction of a data set to more concise set of statistics with no loss of information about the unknown parameter

→ S will be considered a “sufficient” statistics for θ if P(T|S) does not involve θ

Example 10.1.1

A coin is tossed *n* times, find S that sufficient for θ !

Definition 10.2.1

Let $X=(X_1,X_2,\dots,X_n)$ have joint pdf $f(x,\theta)$ and let $S=(S_1,S_2,\dots,S_n)$ be a k-dimensional statistics. Then $S_1,S_2,\dots,S_n$ is a set of jointly sufficient statistics for θ if for any other vector of statistics, T, the conditional pdf of T given $S=s$, denoted by $f_{T|s}(t)$ does not depend on θ. In the one-dimensional case, say that S is a sufficient statistic for θ

Def. 10.2.2

A set of statistics is called a minimal sufficient set if the member of the set are jointly sufficient for the parameters and if they are function of every other set of jointly sufficient statistics

Example 10.2.1

Consider $X_i \sim EXP(\theta)$

Then find *S* that sufficient for θ

Th. 10.2.1 Factorization Criterion

If $X_1, X_2, ..., X_n$ have joint pdf $f(x_1, x_2, ..., x_n)$ and if $S = (S_1, S_2, ..., S_k)$ are jointly sufficient for θ i.o.i

$$f(x_1, x_2, ..., x_n; \theta) = g(s; \theta) h(x_1, x_2, ..., x_n)$$

where $g(s;\theta)$ doesnt depend on $x_1, x_2, ..., x_n$ except through s and $h(x_1, x_2, ..., x_n)$ doesnt involve θ

Example

1. if $X_i \sim BIN(1,\theta)$ then find s
2. if $X_i \sim UNIF(0,\theta)$ then find s
3. if $X_i \sim N(\mu,\sigma^2)$ then find s

Do the exercise at Bain, page 352, number 1, 2 and 3 !