Chapter I

Part 3

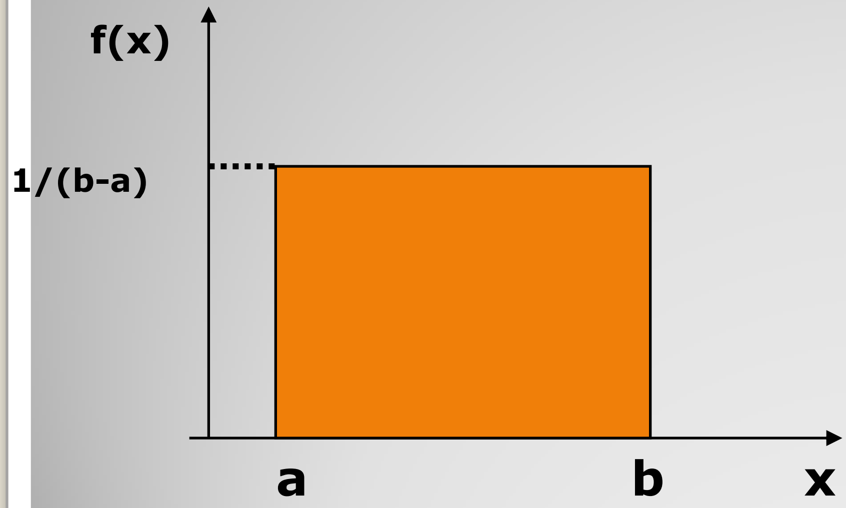
Continuous Distribution

1. Continuous Uniform Distribution

A continuous rV X with pdf

$$f(x;a,b) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

Is a continuous uniform rV or X~UNIF(a,b)



Mean :

$$\mu = \frac{a+b}{2}$$

→ MIDPOINT

Variance :

$$\sigma^2 = \frac{(b-a)^2}{12}$$

→square length I(a,b)

The cumulative distribution function of a continuous uniform random variable is obtained by integration. If $a < x < b$,

$$F(x) = \int_a^x 1/(b-a)\,du = x/(b-a) - a/(b-a)$$

Therefore, the complete description of the cumulative distribution function of a continuous uniform random variable is

$$F(x) = \begin{cases} 0 & x < a \\ (x-a)/(b-a) & a \le x < b \\ 1 & b \le x \end{cases}$$

example

Let X Continuous rV denote the current measured in a thin cooper wire in mA. Assume that the range of X is [0,20mA] and assume that the pdf of X is

$$f(x) = 0.05, \quad 0 \le x \le 20$$

What is the probability that a measurement of current is between 5 and 10 mA?

$$P(5 < X < 10) = \int_{5}^{10} f(x)dx = 5(0.05) = 0.025$$

$$E(X) = 10 \quad V(X) = \frac{20^2}{12} = 33.33$$

2. Gamma Distr

The **gamma function** is

$$\Gamma(r) = \int_0^\infty x^{r-1} e^{-x}\, dx, \quad \text{for } r > 0 \tag{4-19}$$

If X is a **gamma random variable** with parameters λ and r,

$$\mu = E(X) = r/\lambda \quad \text{and} \quad \sigma^2 = V(X) = r/\lambda^2 \tag{4-21}$$

Theorema

$$\Gamma(\kappa) = (\kappa - 1)\Gamma(\kappa - 1), \quad \kappa > 1$$

$$\Gamma(n) = (n-1)!$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

PDF GAMMA FUNCTION

$$X \sim GAM(\theta, \kappa),\ f(x;\theta,\kappa) = \frac{1}{\theta^{\kappa}\Gamma(\kappa)} x^{\kappa-1} e^{-\frac{x}{\theta}}, \quad x > 0$$

$$E[X] = \kappa\theta$$

$$V(X) = \kappa\theta^2$$

3. Normal Distribution (Gaussian)

- The most used model for the distribution of a rV ? why?
- Whenever a random experiment is replicated, the rV that equal the average (total) result over the replicates tends to have a normal distribution as the number of replicates become large → De Moivre → The CLT

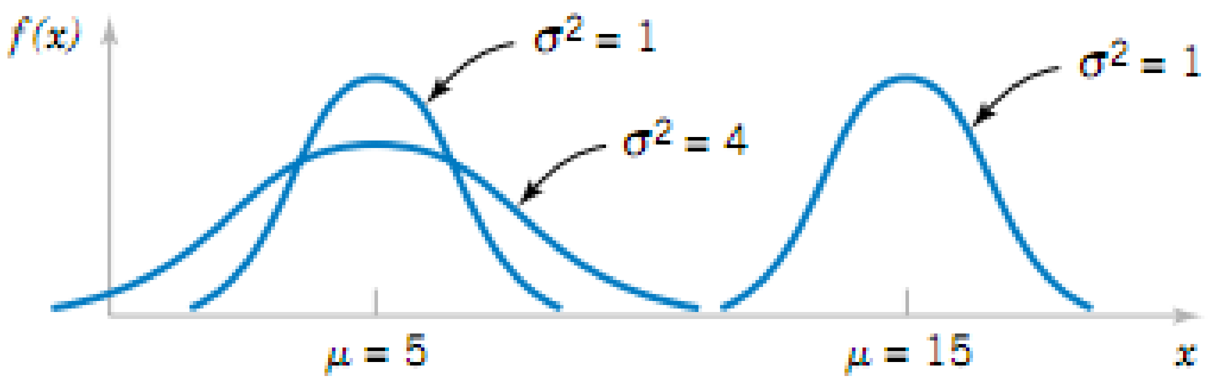
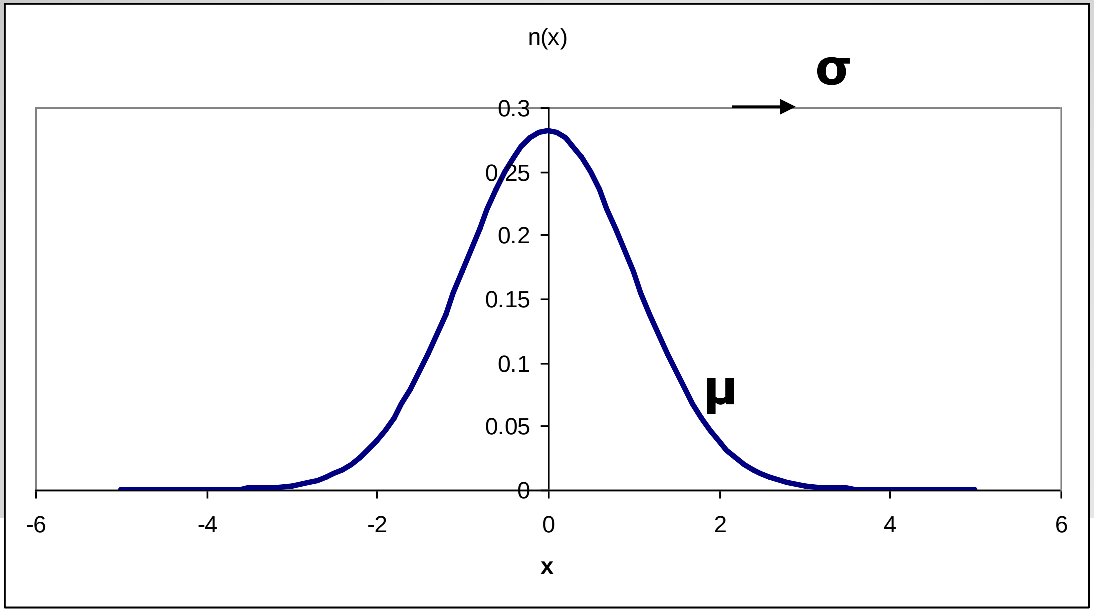


Figure 4-10 Normal probability density functions for selected values of the parameters μ and σ^2.

Normal curve shape



- A Rv X with pdf :

$$f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\ \sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

with parameter $\mu, \quad -\infty < \mu < \infty,$

and $0 < \sigma < \infty,$

$E(X) = \mu, \quad V(X) = \sigma^2$

Notation $N(\mu, \sigma^2)$

Standar Normal PDF

$$\text{If } z = \frac{x-\mu}{\sigma} \text{ then } \Phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, -\infty < z < \infty$$

$$Z \sim N(0,1)$$

$$\Phi(z) = -\Phi(z)$$

$$\Phi'(z) = -z\Phi(z)$$

$$\Phi''(z) = (z^2 - 1)\Phi(z)$$

A normal random variable with

$$\mu = 0 \quad \text{and} \quad \sigma^2 = 1$$

is called a **standard normal random variable** and is denoted as Z.

The cumulative distribution function of a standard normal random variable is denoted as

$$\Phi(z) = P(Z \leq z)$$

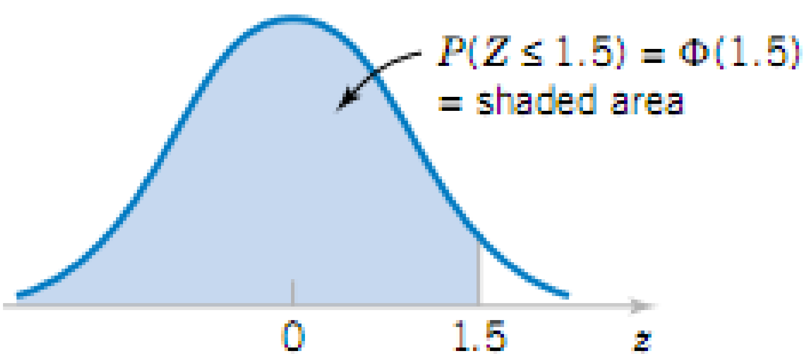
→$\phi(z)$ has unique maximum at $z=0$

→Inflection points at $z=\pm 1$

example

Assume Z is a standard normal random variable.
the form $P(Z \le z)$. The use of Table II to find $P(Z \le 1.5)$ is illustrated in Fig. 4-13. Read down the z column to the row that equals 1.5. The probability is read from the adjacent column, labeled 0.00, to be 0.93319.

The column headings refer to the hundredth's digit of the value of z in $P(Z \le z)$. For example, $P(Z \le 1.53)$ is found by reading down the z column to the row 1.5 and then selecting the probability from the column labeled 0.03 to be 0.93699.



z	0.00	0.01	0.02	0.03
0	0.50000	0.50399	0.50398	0.51197
⋮		⋮		
1.5	0.93319	0.93448	0.93574	0.93699

Other ex

(1) $P(Z > 1.26) = 1 - P(Z \leq 1.26) = 1 - 0.89616 = 0.10384$

(2) $P(Z < -0.86) = 0.19490.$

(3) $P(Z > -1.37) = P(Z < 1.37) = 0.91465$

(4) $P(-1.25 < Z < 0.37).$

$$P(Z < 0.37) = 0.64431 \quad \text{and} \quad P(Z < -1.25) = 0.10565$$

Therefore,

$$P(-1.25 < Z < 0.37) = 0.64431 - 0.10565 = 0.53866$$

Th

If $X \sim N(\mu,\sigma^2)$ then

$$1.\ Z = \frac{X-\mu}{\sigma} \sim N(0,1)$$

$$2.\ F_X(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

If X is a normal random variable with $E(X) = \mu$ and $V(X) = \sigma^2$, the random variable

$$Z = \frac{X - \mu}{\sigma} \tag{4-10}$$

is a normal random variable with $E(Z) = 0$ and $V(Z) = 1$. That is, Z is a standard normal random variable.

Normal Approximation to the Binomial Distribution

If X is a binomial random variable,

$$Z = \frac{X - np}{\sqrt{np(1-p)}} \tag{4-12}$$

is approximately a standard normal random variable. The approximation is good for

$$np > 5 \quad \text{and} \quad n(1-p) > 5$$

Normal approximation to the Poisson Distribution

If X is a Poisson random variable with $E(X) = \lambda$ and $V(X) = \lambda$,

$$Z = \frac{X - \lambda}{\sqrt{\lambda}} \tag{4-13}$$

is approximately a standard normal random variable. The approximation is good for

$$\lambda > 5$$

4. Exponential Distribution

- Let the rV N denote the number of flaws in x mm of wire. If the mean number of flaws is λ per-mm, N has a Poisson distribution with mean λx.
- Now

$$P(X > x) = P(N = 0) = \frac{e^{-\lambda x}(\lambda x)^0}{0!} = e^{-\lambda x}$$

therefore

$$F(x) = P(X \le x) = 1 - e^{-\lambda x}, \quad x \ge 0$$

and

$$f(x) = \lambda e^{-\lambda x}, \quad x \ge 0$$

The random variable X that equals the distance between successive counts of a Poisson process with mean $\lambda > 0$ is an **exponential random variable** with parameter λ. The probability density function of X is

$$f(x) = \lambda e^{-\lambda x} \quad \text{for} \quad 0 \leq x < \infty \tag{4-14}$$

If the random variable X has an exponential distribution with parameter λ,

$$\mu = E(X) = \frac{1}{\lambda} \quad \text{and} \quad \sigma^2 = V(X) = \frac{1}{\lambda^2} \tag{4-15}$$

ex

In a large corporate computer network, user log-ons to the system can be modeled as a Poisson process with a mean of 25 log-ons per hour. What is the probability that there are no log-ons in an interval of 6 minutes?

solution

Let X denote the time in hours from the start of the interval until the first log-on. Then, X has an exponential distribution with $\lambda = 25$ log-ons per hour. We are interested in the probability that X exceeds 6 minutes. Because λ is given in log-ons per hour, we express all time units in hours. That is, 6 minutes = 0.1 hour. The probability requested is shown as the shaded area under the probability density function in Fig. 4-23. Therefore,

$$P(X > 0.1) = \int_{0.1}^{\infty} 25e^{-25x}\,dx = e^{-25(0.1)} = 0.082$$

Other distribution

Possibility new thema about other distribution excluded in MATHSTAT :

- → Erlang distribution
- → Log normal
- → Chi Kuadrat
- → Gumbell
- → Tweedie
- → Rayleight
- → Beta
- → Pearson
- → Cauchy
- → Benford