Chapter 1

Point Estimation part 1
Methods of Estimation

Qualities of Estimator → determined the "best" way to estimate a population parameter.

→**How? unbiasedness, consistency and relative efficiency:**

→An ***unbiased estimator*** of a population parameter is an estimator whose expected value is equal to that parameter.

→An unbiased estimator is said to be ***consistent*** if the difference between the estimator and the parameter grows smaller as the sample size grows larger.

→If there are two unbiased estimators of a parameter, the one whose variance is smaller is said to be ***relatively efficient***.

ESTIMATORS

1. An estimator $\hat{\theta}$ is said to be UNBIASED for the parameter θ if $E\left[\hat{\theta}\right] = \theta$

2. If this equality does'nt hold, $\hat{\theta}$ is said to be a BIASED ESTIMATOR of θ with $BIAS = E\left[\hat{\theta}\right] - \theta$

Definition

A statistic, $T = l(X_1, X_2, \ldots, X_n)$ that is used to estimate the value of $\tau(\theta)$ is called an estimator of $\tau(\theta)$, and an observed value of the statistic, $t = l(x_1, x_2, \ldots, x_n)$ is called an estimate of $\tau(\theta)$

Mostly methods on Point estimation :

→**Moments**

→**Maximum Likelihood**

Review at MathStat 1

- The moment about the origin were defined :

$$\mu'_j\left(\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_k\right) = E\left[X^j\right] \quad j = 1,2\ldots, k$$

1. Method of Moment

Definition 9.2.1 pg 291

If $X_1, X_2, \ldots, X_n$ is a random sample from $f(x; \theta_1, \ldots, \theta_k)$

the first k sample moments are given $M_j^{'} = \dfrac{\sum_{i=1}^{n} X_i^j}{n}, \ j = 1, .., k$

In other words

$$M_j^{'} = \mu_j^{'}\left(\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_k\right) \quad j = 1, 2..., k$$

Example 9.2.1

A r.V sample from a distribution with two unknown parameters the mean μ and the variance σ^2 then find $\hat{\mu}$ and $\hat{\sigma}^2$

Example 9.2.3

A r.V sample from an exponential distribution, $X_i \sim EXP(\theta)$ **then estimate the probability** $p(\theta) = P(X \geq 1)$

2. Method of Maximum Likelihood

Definition 9.2.2

The joint density function of n random variable $X_1, X_2, \ldots, X_n$ evaluated at $x_1, x_2, \ldots, x_n$ say $f(x_1, x_2, \ldots, x_n; \theta)$ is refered to as Likelihood function.

For fixed $x_1, x_2, \ldots, x_n$ the likelihood function is a function of θ and often denoted by $L(\theta)$

If $X_1, X_2, \ldots, X_n$ represent a random sample from $f(x;\theta)$, then

$$L(\theta) = f(x_1;\theta) \cdots f(x_n;\theta)$$

Definition 9.2.3

Let $L(\theta) = f(x_1, \ldots, x_n; \theta), \theta \in \Omega$, be the joint pdf of $X_1, X_2, \ldots, X_n$.

For given set of observations, $(x_1, x_2, \ldots, x_n)$ a value $\hat{\theta}$ in Ω at which

$L(\theta)$ is a maximum likelihood estimate (MLE) of θ is a value of θ that satisfies

$$f(x_1, \ldots, x_n; \hat{\theta}) = \max_{\theta \in \Omega} f(x_1, \ldots, x_n; \theta)$$

Th. Invariance Property

If $\hat{\theta}$ is MLE of θ and if $u(\theta)$ is a function of θ then $u\left(\hat{\theta}\right)$ is an MLE of $u(\theta)$

If $\hat{\theta} = \left(\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_k\right)$ denotes the MLE of $\theta = \left(\theta_1, \theta_2, \ldots, \theta_k\right)$ then the MLE of $\tau = \left(\tau_1(\theta), \tau_2(\theta), \ldots, \tau_r(\theta)\right)$ is

$$\hat{\tau} = \left(\hat{\tau}_1, \hat{\tau}_2, \ldots, \hat{\tau}_r\right) = \left(\hat{\tau}_1(\theta), \hat{\tau}_2(\theta), \ldots, \hat{\tau}_r(\theta)\right) \text{for} \quad 1 \le r \le k$$

Example

1. If $X_i \sim POI(\theta)$ then find $\hat{\theta}$
2. If $X_i \sim EXP(\theta)$ then find $\hat{\theta}$
3. If $X_i \sim N(\mu, \sigma^2)$ then find $\hat{\mu}, \hat{\sigma}^2$