BAB 2
ANAVA 2 JALAN

→Merupakan pengembangan dari ANAVA 1 Jalan

Jika pada ANAVA 1 jalan → 1 Faktor

Jika pada ANAVA 2 jalan → 2 Faktor

Model Linier

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij} \begin{cases} i = 1,\ldots,a \\ j = 1,\ldots,n \end{cases}$$

Anava 1 jalan

$$y_{ijk} = \mu + \tau_i + \beta_j + \varepsilon_{ijk} \begin{cases} i = 1,2,\ldots,a \\ j = 1,2,\ldots,b \\ k = 1,2,\ldots,n \end{cases}$$

Anava 2 jalan tanpa interaksi

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \varepsilon_{ijk} \begin{cases} i = 1,2,\ldots,a \\ j = 1,2,\ldots,b \\ k = 1,2,\ldots,n \end{cases}$$

Anava 2 jalan dengan interaksi

Contoh di bidang industri

Seorang eksperimenter ingin mengetahui pengaruh 3 lempeng (A) pada 3 tingkat suhu (B) 15, 70 dan 125 derajat F. 4 baterai dites pada tiap kombinasi antara faktor lempeng dan suhu.

Table 5-1 Life (in hours) Data for the Battery Design Example

Material Type	Temperature (°F)					
	15		70		125	
1	130	155	34	40	20	70
	74	180	80	75	82	58
2	150	188	136	122	25	70
	159	126	106	115	58	45
3	138	110	174	120	96	104
	168	160	150	139	82	60

Pertanyaan yang muncul adalah :

1. Apakah faktor lempeng berpengaruh terhadap daya hidup baterai ?

2. Apakah faktor suhu berpengaruh terhadap daya hidup baterai?

3. Apakah jenis lempeng material memberikan daya hidup baterai yang seragam tanpa tergantung dari suhu?

Pertanyaan kedua inilah yang mengindikasikan kita menggunakan rancangan faktorial 2 faktor (2 jalan)
→ adanya interaksi antara faktor lempeng (A) dengan faktor suhu (B)

- Contoh di atas merupakan dari rancangan faktorial (anava 2 jalan).

Jika y_{ijk} variabel respon saat faktor A pada tingkat ke-i $(i = 1,2,...,a)$ dan faktor B pada tingkat ke-j $(j = 1,2,...,b)$ untuk replikasi ke-k $(k = 1,2,...,n)$ maka model linier nya adalah:

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \varepsilon_{ijk} \begin{cases} i = 1,2,...,a \\ j = 1,2,...,b \\ k = 1,2,...,n \end{cases}$$

Asumsi model Efek Tetap

1. $\sum_{i=1}^{a} \tau_i = 0, \sum_{j=1}^{b} \beta_j = 0, \sum_{i=1}^{a} (\tau\beta)_{ij} = \sum_{j=1}^{b} (\tau\beta)_{ij} = 0$
2. $\varepsilon_{ijk} \sim NID\left(0, \sigma^2\right)$

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \varepsilon_{ijk} \begin{cases} i = 1,2,\ldots,a \\ j = 1,2,\ldots,b \\ k = 1,2,\ldots,n \end{cases}$$

$$y_{ijk} = \mu_{ij} + \varepsilon_{ijk}$$

$$\mu_{ij} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij}$$

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \varepsilon_{ijk}$$

Jika $\mu_{ij} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij}$

maka $y_{ijk} = \mu_{ij} + \varepsilon_{ijk}$

Jadi estimasi dari y adalah

$$E[y_{ijk}] = E[\mu_{ij} + \varepsilon_{ijk}]$$
$$= E[\mu_{ij}] + E[\varepsilon_{ijk}]$$
$$\hat{y}_{ijk} = \hat{\mu}_{ij}$$

Estimasi dari μ_{ij}

$$E\left[y_{ij}\right] = E\left[\mu_{ij} + \varepsilon_{ij}\right] = E\left[\mu_{ij}\right] = \hat{\mu}_{ij} \Rightarrow \hat{\mu}_{ij} \text{ ???}$$

$$Q_{ij} = \sum_{k}^{n} \left(y_{ijk} - \mu_{ij}\right)^2$$

$$\frac{dQ_{ij}}{d\mu_{ij}} = 2\sum_{k}^{n} \left(y_{ijk} - \hat{\mu}_{ij}\right).-1 = 0$$

$$\sum_{k}^{n} y_{ijk} - \sum_{k}^{n} \hat{\mu}_{ij} = 0$$

$$\sum_{k}^{n} y_{ijk} = n\hat{\mu}_{ij}$$

$$\frac{\sum_{k}^{n} y_{ijk}}{n} = \hat{\mu}_{ij}$$

$$\hat{\mu}_{ij} = \frac{y_{ij\bullet}}{n} = \bar{y}_{ij\bullet}$$

Step-step uji Anava 2 jalan

1. Susun Hipotesis

$H_{0A} : \tau_1 = \tau_2 = \cdots = \tau_a = 0$

$H_{1A} : \text{paling tidak ada satu } \tau_i \neq 0$

$H_{0B} : \beta_1 = \beta_2 = \cdots = \beta_b = 0$

$H_{1B} : \text{paling tidak ada satu } \beta_j \neq 0$

$H_{0AB} : (\tau\beta)_{ij} = 0, \forall ij$

$H_{1AB} : \text{paling tidak ada satu } (\tau\beta)_{ij} \neq 0$

2.Pilih tingkat signifikansi

3. Susun Tabel ANAVA 2 Jalan

Factor A	Factor B: 1	2	...	b
1	$y_{111}, y_{112}, \ldots, y_{11n}$	$y_{121}, y_{122}, \ldots, y_{12n}$		$y_{1b1}, y_{1b2}, \ldots, y_{1bn}$
2	$y_{211}, y_{212}, \ldots, y_{21n}$	$y_{221}, y_{222}, \ldots, y_{22n}$		$y_{2b1}, y_{2b2}, \ldots, y_{2bn}$
⋮				
a	$y_{a11}, y_{a12}, \ldots, y_{a1n}$	$y_{a21}, y_{a22}, \ldots, y_{a2n}$		$y_{ab1}, y_{ab2}, \ldots, y_{abn}$

$$y_{i..} = \sum_{j=1}^{b} \sum_{k=1}^{n} y_{ijk} \qquad \bar{y}_{i..} = \frac{y_{i..}}{bn} \qquad i = 1, 2, \ldots, a$$

$$y_{.j.} = \sum_{i=1}^{a} \sum_{k=1}^{n} y_{ijk} \qquad \bar{y}_{.j.} = \frac{y_{.j.}}{an} \qquad j = 1, 2, \ldots, b$$

$$y_{ij.} = \sum_{k=1}^{n} y_{ijk} \qquad \bar{y}_{ij.} = \frac{y_{ij.}}{n} \qquad \begin{matrix} i = 1, 2, \ldots, a \\ j = 1, 2, \ldots, b \end{matrix}$$

$$y_{...} = \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} y_{ijk} \qquad \bar{y}_{...} = \frac{y_{...}}{abn}$$

Partisi JKT

$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}\left(y_{ijk}-\bar{y}_{\bullet\bullet\bullet}\right)^2$$

$$=\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}\left[\left(\bar{y}_{i\bullet\bullet}-\bar{y}_{\bullet\bullet\bullet}\right)+\left(\bar{y}_{\bullet j\bullet}-\bar{y}_{\bullet\bullet\bullet}\right)+\left(\bar{y}_{ij\bullet}-\bar{y}_{i\bullet\bullet}-\bar{y}_{\bullet j\bullet}+\bar{y}_{\bullet\bullet\bullet}\right)+\left(y_{ijk}-\bar{y}_{ij\bullet}\right)\right]^2$$

$$=\underbrace{bn\sum_{i=1}^{a}\left(\bar{y}_{i\bullet\bullet}-\bar{y}_{\bullet\bullet\bullet}\right)^2}_{JK_A}+\underbrace{an\sum_{j=1}^{b}\left(\bar{y}_{\bullet j\bullet}-\bar{y}_{\bullet\bullet\bullet}\right)^2}_{JK_B}+\underbrace{n\sum_{i=1}^{a}\sum_{j=1}^{b}\left(\bar{y}_{ij\bullet}-\bar{y}_{i\bullet\bullet}-\bar{y}_{\bullet j\bullet}+\bar{y}_{\bullet\bullet\bullet}\right)^2}_{JK_{AB}}+\underbrace{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}\left(y_{ijk}-\bar{y}_{ij\bullet}\right)^2}_{JK_S}$$

$$JK_T = JK_A + JK_B + JK_{AB} + JK_S$$

dengan

$$JK_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{\bullet\bullet\bullet}^2}{abn}$$

no 1

$$JK_A = \frac{1}{bn}\sum_{i=1}^{a} y_{i\bullet\bullet}^2 - \frac{y_{\bullet\bullet\bullet}^2}{abn}$$

no 2

$$JK_B = \frac{1}{an}\sum_{j=1}^{b} y_{\bullet j\bullet}^2 - \frac{y_{\bullet\bullet\bullet}^2}{abn}$$

no 3

$$JK_{\text{Sub total}} = \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij\bullet}^2 - \frac{y_{\bullet\bullet\bullet}^2}{abn}$$

$$JK_{AB} = JK_{\text{Sub total}} - JK_A - JK_B$$

$$JK_S = JK_T - JK_{AB} - JK_A - JK_B$$

Tabel ANAVA

SV	db	JK	RK	F
A	a-1	JKA	RKA=JKA/dbA	FA
B	b-1	JKB	RKB=JKB/dbB	FB
AB	(a-1)(b-1)	JK(AB)	RK(AB)=JK(AB)/db(AB)	FAB
Sesatan	ab(n-1)	JKS	RKS=JKS/db(S)	
Total	abn-1	JKT		

Contoh soal di atas

Material Type	Temperature (°F) 15			70			125			$y_{i..}$
1	130 74	155 180	(539)	34 80	40 75	(229)	20 82	70 58	(230)	998
2	150 159	188 126	(623)	136 106	122 115	(479)	25 58	70 45	(198)	1300
3	138 168	110 160	(576)	174 150	120 139	(583)	96 82	104 60	(342)	1501
$y_{.j.}$		1738			1291			770		3799 = $y_{...}$

erature (°F)				
70		125		$y_{i..}$
40 (229)	20	70 (230)		998
75	82	58		
122 (479)	25	70 (198)		1300
115	58	45		
120 (583)	96	104 (342)		1501
139	82	60		
1291		770		$3799 = y_{...}$

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{abn}$$

$$= (130)^2 + (155)^2 + (74)^2 + \cdots + (60)^2 - \frac{(3799)^2}{36} = 77{,}646.97$$

$$SS_{\text{Material}} = \frac{1}{bn}\sum_{i=1}^{a} y_{i..}^2 - \frac{y_{...}^2}{abn}$$

$$= \frac{1}{(3)(4)}\left[(998)^2 + (1300)^2 + (1501)^2\right] - \frac{(3799)^2}{36} = 10{,}683.72$$

$$SS_{\text{Temperature}} = \frac{1}{an}\sum_{j=1}^{b} y_{.j.}^2 - \frac{y_{...}^2}{abn}$$

$$= \frac{1}{(3)(4)}\left[(1738)^2 + (1291)^2 + (770)^2\right] - \frac{(3799)^2}{36} = 39{,}118.72$$

$$SS_{\text{Interaction}} = \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij.}^2 - \frac{y_{...}^2}{abn} - SS_{\text{Material}} - SS_{\text{Temperature}}$$

$$= \frac{1}{4}\left[(539)^2 + (229)^2 + \cdots + (342)^2\right] - \frac{(3799)^2}{36} - 10{,}683.72$$

$$- 39{,}118.72 = 9613.78$$

$$SS_E = SS_T - SS_{\text{Material}} - SS_{\text{Temperature}} - SS_{\text{Interaction}}$$

$$= 77{,}646.97 - 10{,}683.72 - 39{,}118.72 - 9613.78 = 18{,}230.75$$

Tabel ANAVA

Analysis of Variance for Battery Life Data

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Material types	10,683.72	2	5,341.86	7.91
Temperature	39,118.72	2	19,559.36	28.97
Interaction	9,613.78	4	2,403.44	3.56
Error	18,230.75	27	675.21	
Total	77,646.97	35		

- Tolak HA karena F=7.91> F(0.05,2,27)=3.35. Jadi *material types* (jenis lempeng) berpengaruh terhadap daya hidup baterai
- Tolak HB karena F=28.97 > F(0.05,2,27)=3.35. jadi *temperature* (suhu) berpengaruh terhadap daya hidup baterai
- Tolak HAB karena FAB=3.56 > F(0.05,4,27)=2.73. Jadi faktor interaksi berpengaruh terhadap daya hidup baterai. D.K.L jenis lempeng material tergantung dari suhu terhadap daya hidup baterai

Plot Interaksi antara A dan B