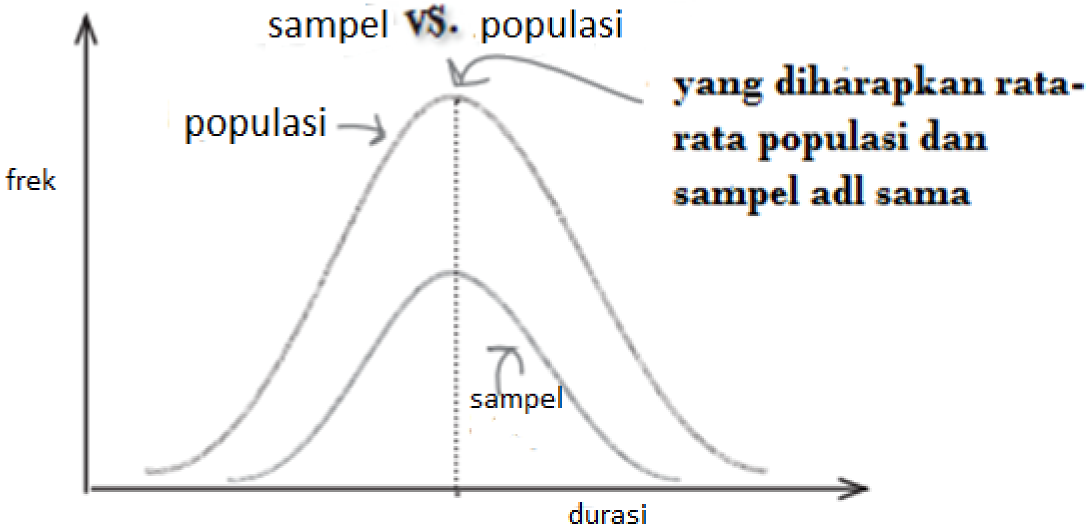
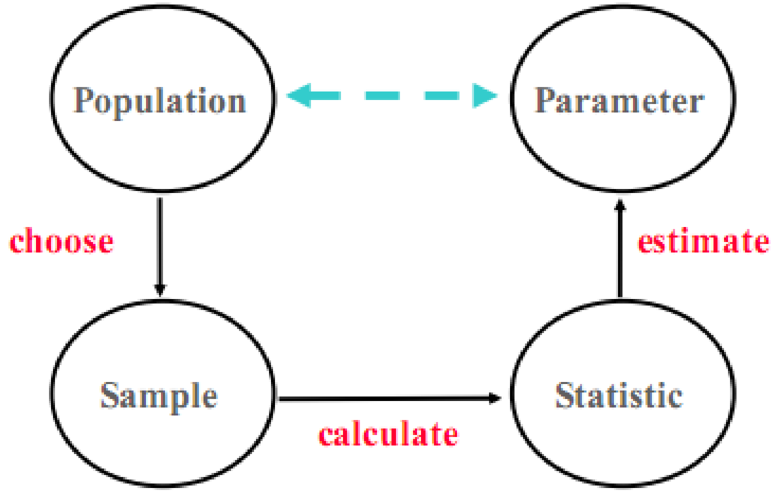
BAB 3

ESTIMASI

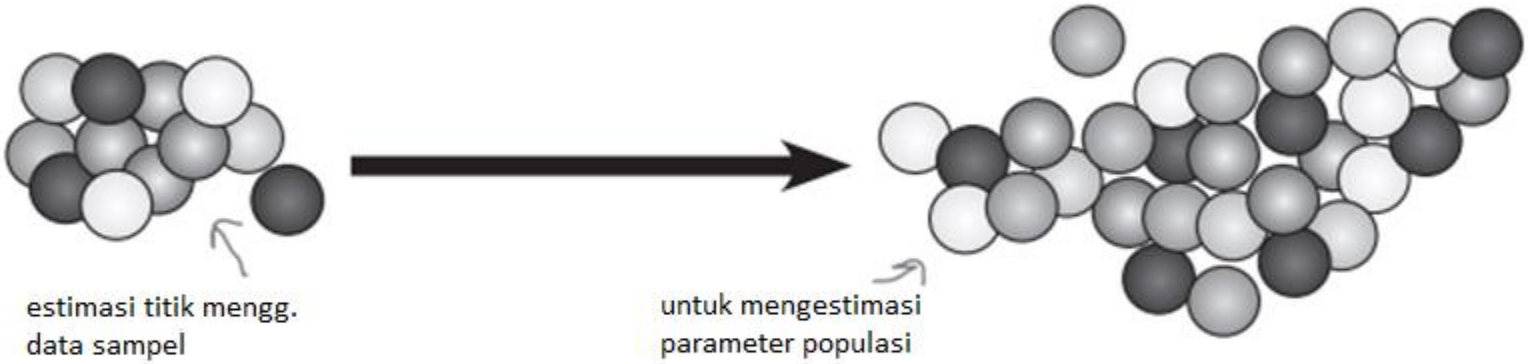
Ilustrasi



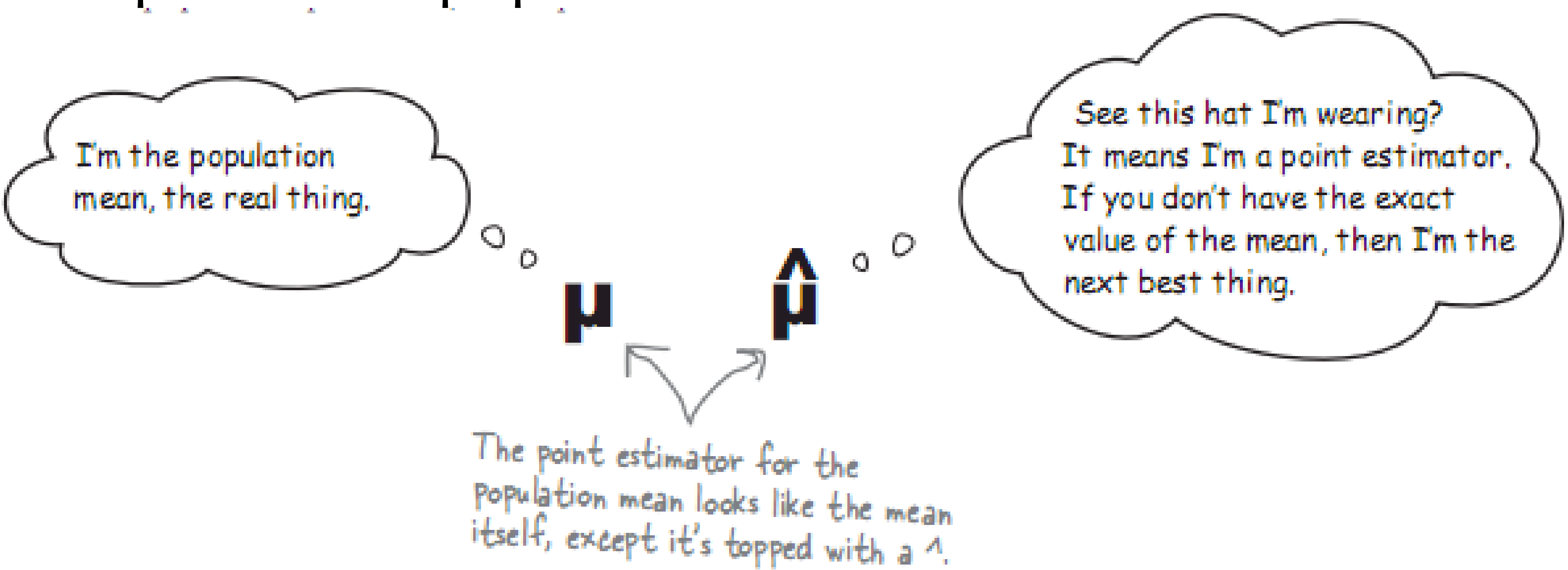


- Mungkin rata-ratanya tidaklah sama tapi estimasi terbaik dapat ditentukan
- Rata-rata sampel→ estimator titik untuk rata-rata populasi

  →Artinya jika dihitung sampel data akan mengestimasi rata-rata populasi



- Estimator titik dapat menghasilkan nilai pendekatan suatu parameter populasi



A **point estimate** of some population parameter θ is a single numerical value $\hat{\theta}$ of a statistic $\hat{\Theta}$. The statistic $\hat{\Theta}$ is called the **point estimator**.

Estimasi titik untuk rata-rata

$\bar{x}$ is the mean of the sample.

$$\bar{x} = \frac{\Sigma x}{n}$$

Add together the numbers in the sample, and divide by how many there are.

We estimate the mean of the population...

$$\hat{\mu} = \bar{x}$$

...using the mean of the sample.

Misalkan variabel random X berdistribusi Normal dengan rata-rata μ tidak diketahui. Misalkan diambil x1=25, x2=30, x3=29 dan x4=31, maka dapat diestimasi rata-ratanya adalah

$$\bar{x} = \frac{25 + 30 + 29 + 31}{4} = 28.75$$

Estimasi titik untuk variansi

Population variance

$$\sigma^2 = \frac{\Sigma(x - \mu)^2}{n}$$

Population mean

Size of the population

Point estimator for the population variance

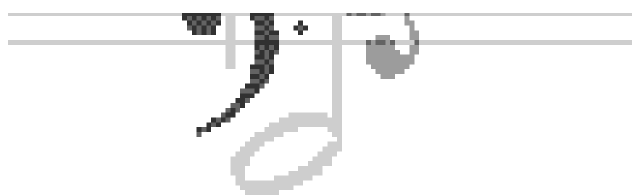
$$\hat{\sigma}^2 = s^2$$

Estimator for the population variance

$$\hat{\sigma}^2 = \frac{\Sigma(x - \bar{x})^2}{n - 1}$$

Take each item in the sample, subtract the sample mean, square the result, then add the lot together.

Divide by the number in the sample minus 1.



latihan

Tentukan estimasi titik untuk rata-rata dan variansi untuk data :

61.9   62.6   63.3   64.8   65.1   66.4   67.1   67.2   68.7   69.9

Estimasi untuk Proporsi

- Jika X menggambarkan jumlah **<u>sukses</u>** dalam suatu populasi maka X mengikuti distribusi Bin(n,p)
- Misal akan diestimasi rata-rata populasi ???
- Estimasi rata-rata populasi → rata-rata sampel
- Proporsi sukses dalam populasi → proporsi sukses dalam sampel

Point estimator for the proportion of successes in the population

$$\hat{p} = p_s$$

Proportion of successes in the sample

dengan

$$p_s = \frac{\text{number of successes}}{\text{number in sample}}$$

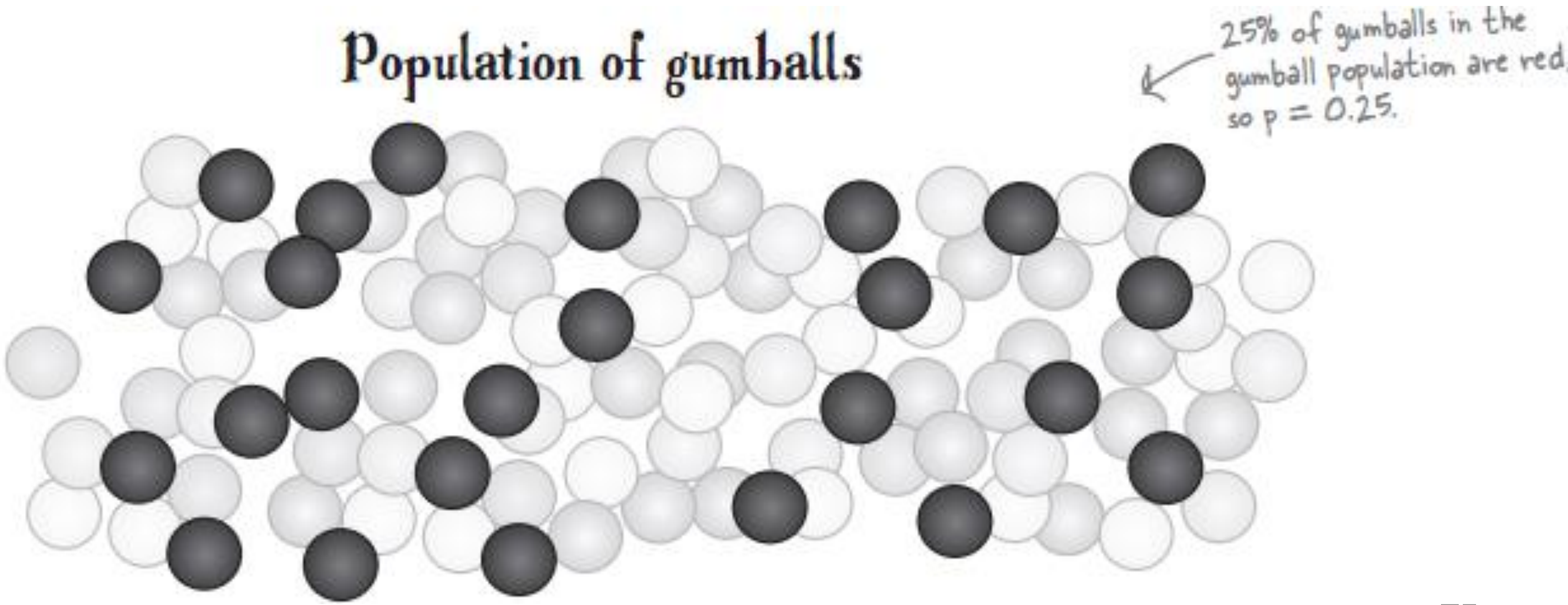
contoh

- Misal diteliti hobi anak-anak terhadap game online di suatu daerah XZ. Diambil sampel sebanyak 40 anak, dan ternyata 32 anak menyukai game online

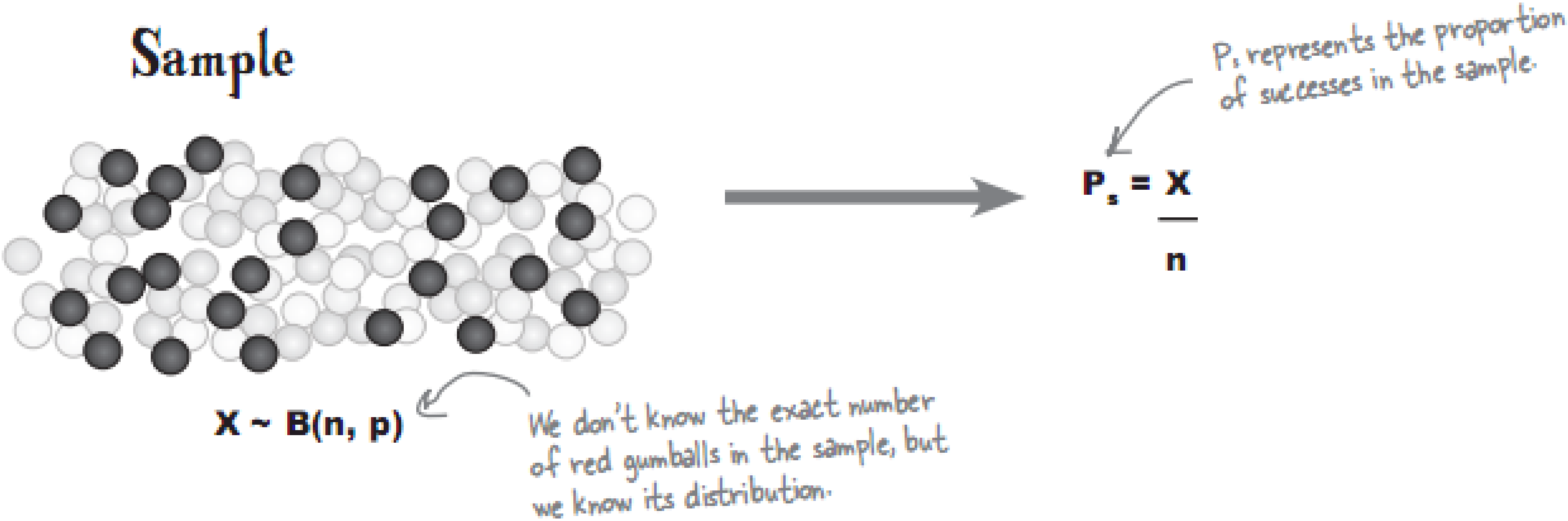
  → Ps=0.8 ; estimasi titik untuk proporsi sukses (anak menyukai game online) dalam populasi adalah 0.8

Distribusi sampling dari proporsi

- Bagaimana menentukan distribusi dari sampel proporsi?
- Contoh

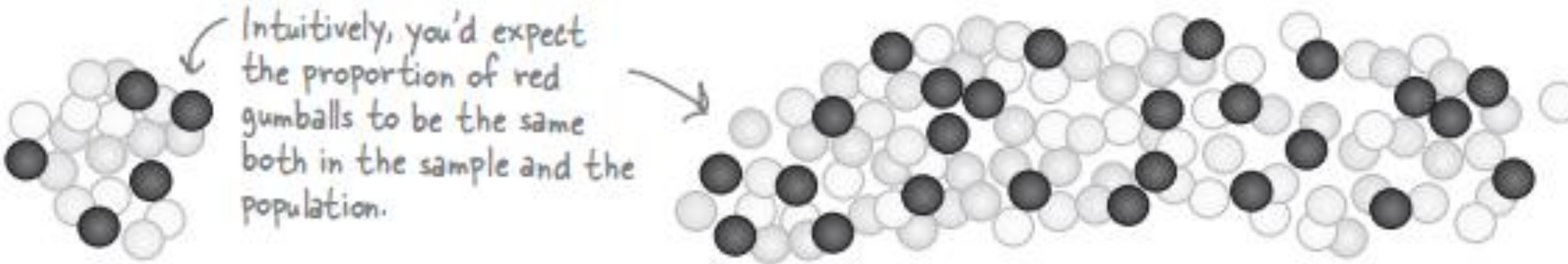


- Jika X menggambarkan jumlah red gumball dalam sampel maka X~Bin(n,p). Misal n=100, p=0.25
- Proporsi red gumball dalam sampel tergantung dari X yaitu jumlah red gumball dalam sampel



- Distribusi dari sampel proporsi dapat ditentukan dengan menggunakan **distribusi sampling proporsi** atau distribusi dari Ps

Ekspektasi & Variansi dari Ps?



$$E(P_s) = E\left(\frac{X}{n}\right)$$

$$= \frac{E(X)}{n}$$

$$= \frac{\cancel{n}p}{\cancel{n}} \longleftarrow E(X) = np$$

$$= p$$

$$Var(P_s) = Var\left(\frac{X}{n}\right)$$

$$= \frac{Var(X)}{n^2}$$

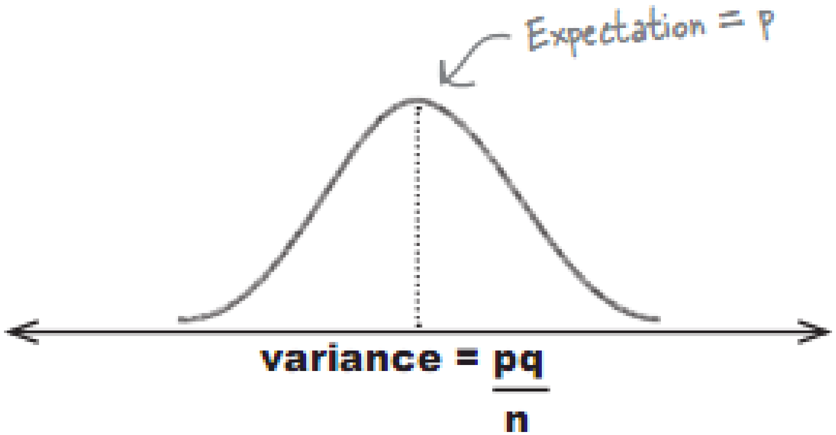
This comes from $Var(aX) = a^2Var(X)$. In this case, $a = 1/n$.

$$= \frac{\cancel{n}pq}{n^{\cancel{2}}} \longleftarrow Var(X) = npq$$

$$= \frac{pq}{n}$$

So…

- Distribusi Ps tergantung dari n…

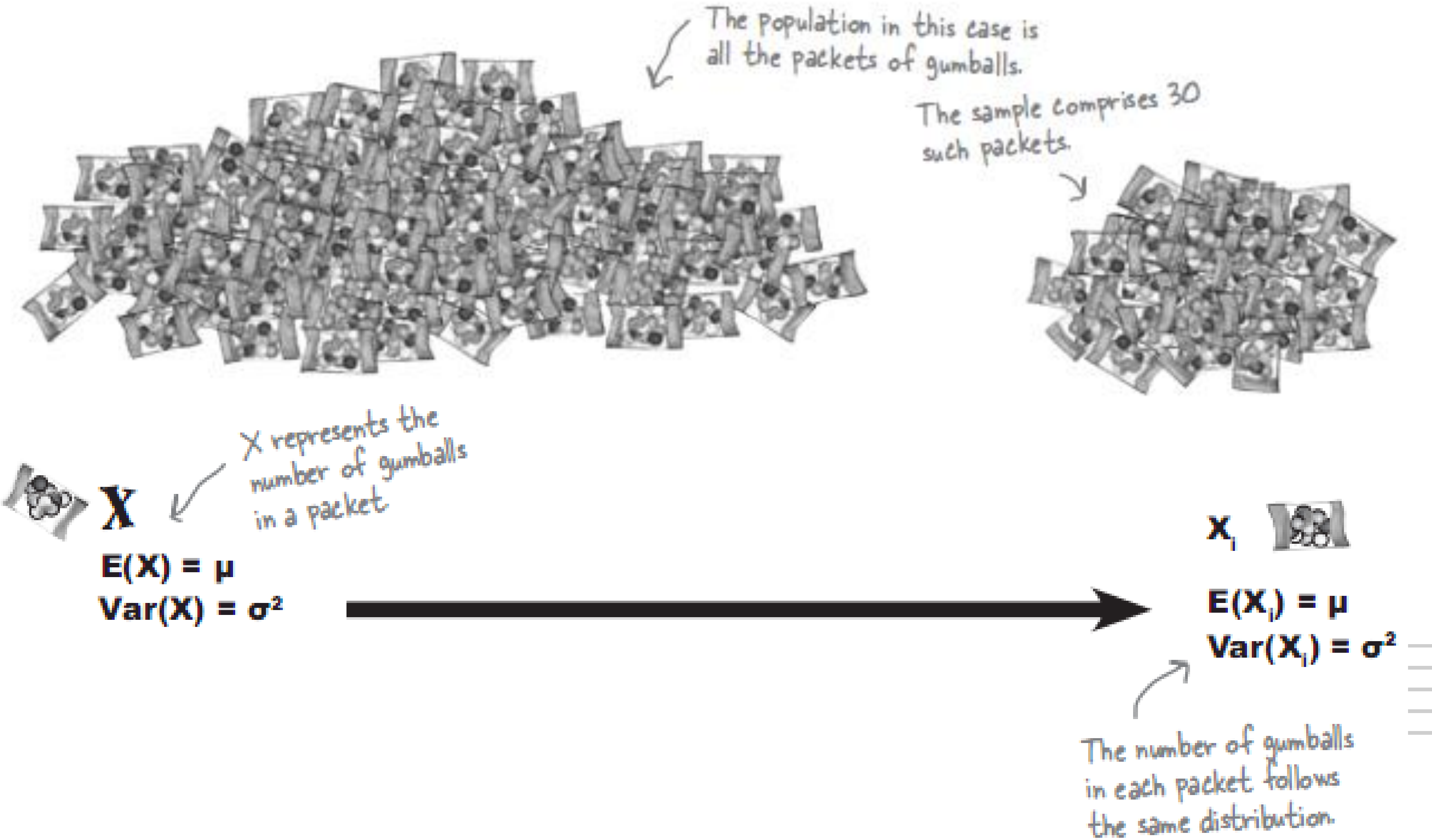


- Jika n bertambah besar (ukuran besar > 30) maka distribusi Ps akan mengikuti distribusi Normal, atau

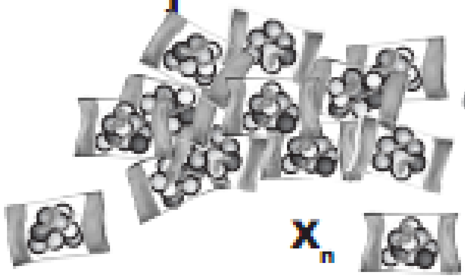
$$P_s \sim N\left(p, \frac{pq}{n}\right)$$

Distribusi sampling untuk rata-rata

- contoh



Sample of X



X_1 X_n

Each X_i is an independent observation of X, so each packet has the same expectation and variance for the number of gumballs.

$E(X_1) = \mu$
$Var(X_1) = \sigma^2$

$E(X_n) = \mu$
$Var(X_n) = \sigma^2$

This is the mean of the sample, the mean number of gumballs in the packets.

$$\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n}$$

Ekspektasi $\bar{X}$

$$E(\bar{X}) = E\left(\frac{X_1 + X_2 + \dots + X_n}{n}\right)$$

These two expressions are the same, just written in a different way.

$$= E\left(\frac{1}{n}X_1 + \frac{1}{n}X_2 + \dots + \frac{1}{n}X_n\right)$$

$$= E\left(\frac{1}{n}X_1\right) + E\left(\frac{1}{n}X_2\right) + \dots + E\left(\frac{1}{n}X_n\right)$$

We can split this into n separate expectations because $E(X + Y) = E(X) + E(Y)$.

Every expectation includes 1/n, so we can take it out of the expression. This comes from $E(aX) = aE(X)$.

$$= \frac{1}{n}\left(E(X_1) + E(X_2) + \dots + E(X_n)\right)$$

Variansi $\bar{x}$?

$$\mathrm{Var}(\bar{X}) = \mathrm{Var}\left(\frac{X_1 + X_2 + \dots + X_n}{n}\right)$$

$$= \mathrm{Var}\left(\boxed{\frac{1}{n}X_1 + \frac{1}{n}X_2 + \dots + \frac{1}{n}X_n}\right)$$

$$= \mathrm{Var}\left(\boxed{\frac{1}{n}X_1}\right) + \mathrm{Var}\left(\boxed{\frac{1}{n}X_2}\right) + \dots + \mathrm{Var}\left(\boxed{\frac{1}{n}X_n}\right)$$

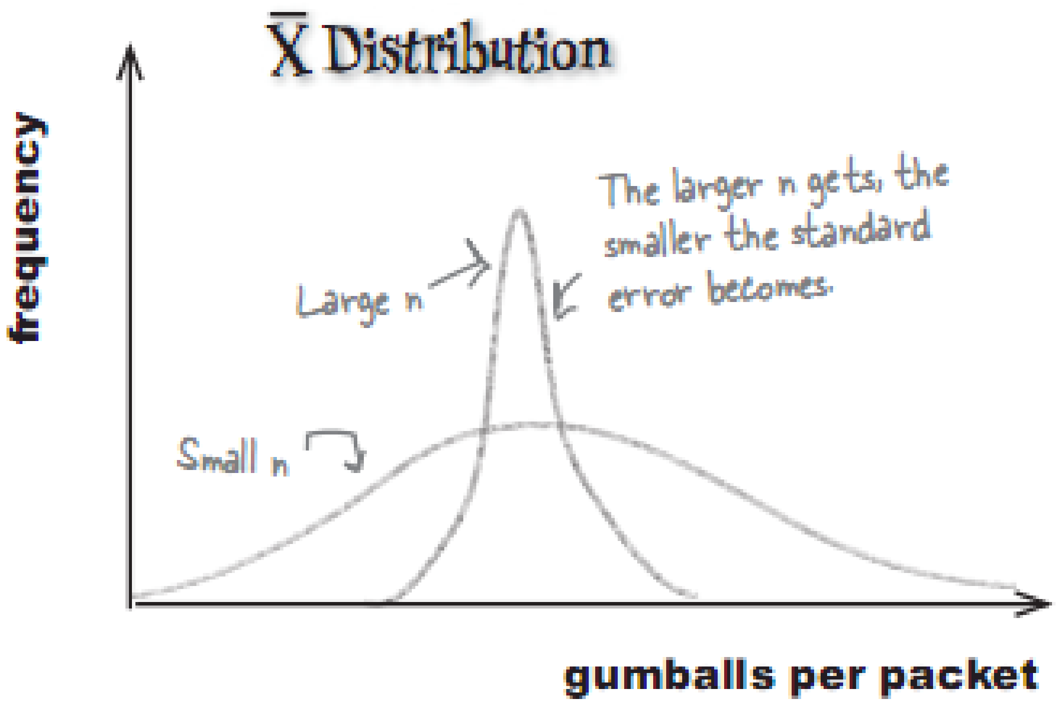
$$= \boxed{\left(\frac{1}{n}\right)^2} (\mathrm{Var}(X_1) + \mathrm{Var}(X_2) + \dots + \mathrm{Var}(X_n))$$

$$= \frac{1}{n^2} \left(\boxed{\sigma^2 + \sigma^2 + \dots + \sigma^2}\right.$$

$$= n \times \frac{1}{n^2} \sigma^2$$

$$= \boxed{\frac{\sigma^2}{n}}$$

So…



If X ~ N(μ, σ²), then $\bar{X}$ ~ N(μ, σ²/n)

These are the mean and variance of $\bar{X}$ that we found earlier.

Teorema Limit Pusat (CLT)

Jika sampel diambil dari suatu populasi dg distribusi non normal dan ukuran sampel besar maka rata-rata X diaproksimasikan berdistribusi Normal dengan rata-rata dan variansi populasi adalah μ, σ^2

$$\bar{X} \sim N(\mu, \sigma^2/n)$$

This is the mean and variance of $\bar{X}$.

Latihan

Cari distribusi rata-rata dari X yang berdistribusi :

a. X~POI(λ)

b. X~Bin(n,p)

Latihan

7-12. Data on oxide thickness of semiconductors are as follows: 425, 431, 416, 419, 421, 436, 418, 410, 431, 433, 423, 426, 410, 435, 436, 428, 411, 426, 409, 437, 422, 428, 413, 416.

(a) Calculate a point estimate of the mean oxide thickness for all wafers in the population.

(b) Calculate a point estimate of the standard deviation of oxide thickness for all wafers in the population.

(c) Calculate the standard error of the point estimate from part (a).

(d) Calculate a point estimate of the median oxide thickness for all wafers in the population.

(e) Calculate a point estimate of the proportion of wafers in the population that have oxide thickness greater than 430 angstrom.