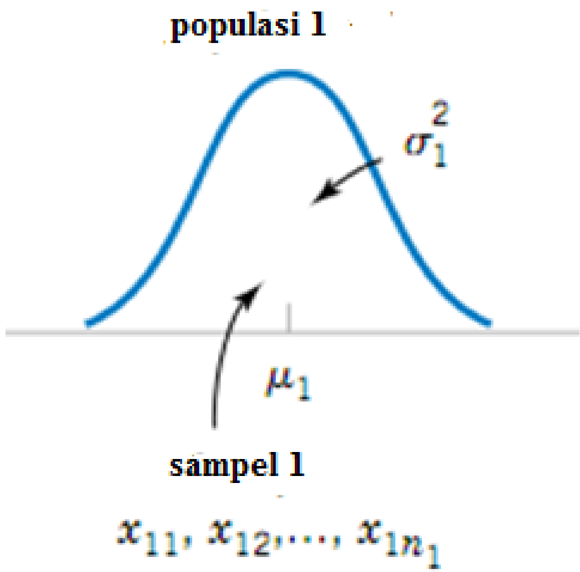
Uji Hipotesis

Dua populasi

Uji hipotesis untuk dua rata-rata

Misal variansi diketahui



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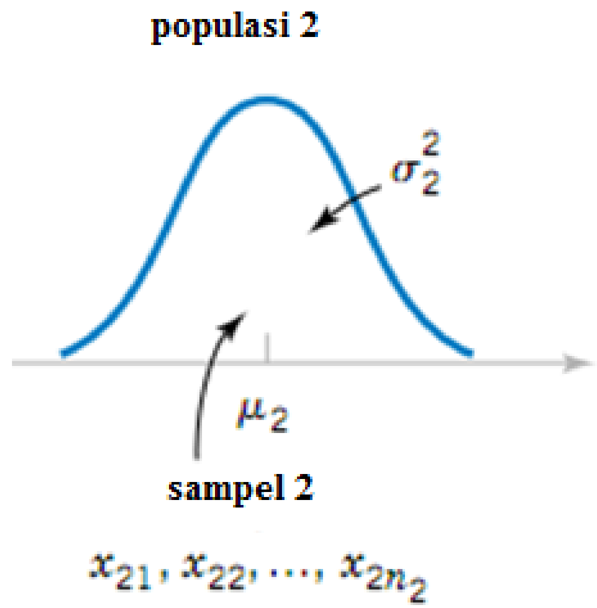

- Asumsi

1. Sampel 1 dan sampel adalah sampel random
2. Populasi 1 dan populasi 2 independen dan normal

Prosedur, variansi tdk diketahui

$\text{Kasus1}: \sigma_1^2 = \sigma_2^2 = \sigma^2$

1. Susun Hipotesis

$$a.\ H_0 : \mu_1 - \mu_2 = \Delta_0$$
$$H_1 : \mu_1 - \mu_2 \neq \Delta_0$$
$$b.\ H_0 : \mu_1 - \mu_2 = \Delta_0$$
$$H_1 : \mu_1 - \mu_2 > \Delta_0$$
$$c.\ H_0 : \mu_1 - \mu_2 = \Delta_0$$
$$H_1 : \mu_1 - \mu_2 < \Delta_0$$

ii. Pilih tingkat signifikansi α

iii. Uji Statistika

$$T_0 = \frac{\overline{X}_1 - \overline{X}_2 - \Delta_0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

dengan

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

iv. Daerah kritis

Tolak H0 jika :

$$a.\ H_0 : \mu_1 - \mu_2 = \Delta_0$$
$$H_1 : \mu_1 - \mu_2 \neq \Delta_0$$

→ $t_0 > t_{\frac{\alpha}{2}, n_1+n_2-2}$ atau $t_0 < -t_{\frac{\alpha}{2}, n_1+n_2-2}$

$$b.\ H_0 : \mu_1 - \mu_2 = \Delta_0$$
$$H_1 : \mu_1 - \mu_2 > \Delta_0$$

→ $t_0 > t_{\alpha, n_1+n_2-2}$

$$c.\ H_0 : \mu_1 - \mu_2 = \Delta_0$$
$$H_1 : \mu_1 - \mu_2 < \Delta_0$$

→ $t_0 < -t_{\alpha, n_1+n_2-2}$

contoh

No	katalis 1	katalis 2
1	91.50	89.19
2	94.18	90.95
3	92.18	90.46
4	95.39	93.21
5	91.79	97.19
6	89.07	97.04
7	94.72	91.07
8	89.21	92.75
	$\bar{x}_1 = 92.255$	$\bar{x}_2 = 92.733$
	$s_1 = 2.39$	$s_2 = 2.98$

Misalkan ingin diketahui apakah rerata kecepatan reaksi dari katalis 1 berbeda dengan katalis 2? (asumsi variansi populasi tidak diketahui)

i. $H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

ii. Dipilih tingkat signifikansi 5%

iii. $t_0 = \dfrac{\bar{x}_1 - \bar{x}_2 - 0}{s_p \sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$

$$= \frac{92,255 - 92,733}{2,7\sqrt{\frac{1}{8} + \frac{1}{8}}} = -0,35$$

$$S_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$= \frac{(7)(2,39)^2 + (7)(2,98)^2}{8 + 8 - 2} = 7,3$$

$$S_p = 2,7$$

iv. Tolak H_0 jika $t_0 > t_{0,025;14} = 2,145$ atau $t_0 < -t_{0,025;14} = -2,145$

Karena $t_0 = -0,35 > -t_{0,025;14} = -2,145$ maka H_0 tidak ditolak

d.k.l kecepatan rerata katalis 1 dan katalis 2 pada tingkat kepercayaan 95% tidak berbeda signifikan

Prosedur, variansi tdk diketahui

$\text{Kasus } 2: \sigma_1^2 \neq \sigma_2^2$

iii. Uji statistika

$$T_0^* = \frac{\overline{X}_1 - \overline{X}_2 - \Delta_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

Dibandingkan dengan distribusi t dengan db v,

$$S_p^2 = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{S_2^2}{n_2}\right)^2}{n_2 - 1}}$$

Contoh 2

Metro Phoenix ($\bar{x}_1 = 12.5, s_1 = 7.63$)	Rural Arizona ($\bar{x}_2 = 27.5, s_2 = 15.3$)
Phoenix, 3	Rimrock, 48
Chandler, 7	Goodyear, 44
Gilbert, 25	New River, 40
Glendale, 10	Apachie Junction, 38
Mesa, 15	Buckeye, 33
Paradise Valley, 6	Nogales, 21
Peoria, 12	Black Canyon City, 20
Scottsdale, 25	Sedona, 12
Tempe, 15	Payson, 1
Sun City, 7	Casa Grande, 18

Misal ingin diketahui rerata konsentrasi arsenik dari dua daerah tersebut apakah berbeda atau kah tidak?

i. $H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

$$v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} = 13,2 = 13$$

ii. Dipilih tingkat signifikansi 5%

iii. $$t^*_0 = \frac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{12,5 - 27,5}{\sqrt{\frac{(7,63)^2}{10} + \frac{(15,3)^2}{10}}} = -2,77$$

iv. Tolak H_0 jika $t^*_0 > t_{0,025;13} = 2,160$ atau $t_0 < -t_{0,025;13} = -2,160$

Karena $t_0 = -2,77 < -t_{0,025;14} = -2,16$ maka H_0 ditolak

Prosedur , variansi diketahui

iii. Uji statistika

$$Z_0 = \frac{\overline{X}_1 - \overline{X}_2 - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

iv. Daerah kritis : tolak H0 jika

$$a.\ H_0 : \mu_1 - \mu_2 = \Delta_0$$
$$H_1 : \mu_1 - \mu_2 \neq \Delta_0$$

→ $z_0 > z_{\frac{\alpha}{2}}$ atau $z_0 < -z_{\frac{\alpha}{2}}$

$$b.\ H_0 : \mu_1 - \mu_2 = \Delta_0$$
$$H_1 : \mu_1 - \mu_2 > \Delta_0$$

→ $z_0 > z_\alpha$

$$c.\ H_0 : \mu_1 - \mu_2 = \Delta_0$$
$$H_1 : \mu_1 - \mu_2 < \Delta_0$$

→ $z_0 < -z_\alpha$

Uji proporsi dua populasi

i. Susun hipotesis

$$a.\ H_0 : p_1 = p_2$$
$$H_1 : p_1 \neq p_2$$
$$b.\ H_0 : p_1 = p_2$$
$$H_1 : p_1 > p_2$$
$$c.\ H_0 : p_1 = p_2$$
$$H_1 : p_1 < p_2$$

ii. Pilih tingkat signifikansi α

iii. Statistika uji

$$Z_0 = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{P}(1-\hat{P})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

iv. Daerah kritis

Tolak H0 jika

$a. H_0 : p_1 = p_2$

$H_1 : p_1 \neq p_2$ ⟹ $z_0 > z_{\frac{\alpha}{2}}$ atau $z_0 < -z_{\frac{\alpha}{2}}$

$b. H_0 : p_1 = p_2$

$H_1 : p_1 > p_2$ ⟹ $z_0 > z_\alpha$

$c. H_0 : p_1 = p_2$

$H_1 : p_1 < p_2$ ⟹ $z_0 < -z_\alpha$

example

1. The parameters of interest are p_1 and p_2, the proportion of patients who improve following treatment with St. John's Wort (p_1) or the placebo (p_2).
2. H_0: $p_1 = p_2$
3. H_1: $p_1 \neq p_2$
4. $\alpha = 0.05$
5. The test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $\hat{p}_1 = 27/100 = 0.27$, $\hat{p}_2 = 19/100 = 0.19$, $n_1 = n_2 = 100$, and

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{19 + 27}{100 + 100} = 0.23$$

6. Reject H_0: $p_1 = p_2$ if $z_0 > z_{0.025} = 1.96$ or if $z_0 < -z_{0.025} = -1.96$.
7. Computations: The value of the test statistic is

$$z_0 = \frac{0.27 - 0.19}{\sqrt{0.23(0.77)\left(\frac{1}{100} + \frac{1}{100}\right)}} = 1.35$$

8. Conclusions: Since $z_0 = 1.35$ does not exceed $z_{0.025}$, we cannot reject the null hypothesis. Note that the P-value is $P \simeq 0.177$. There is insufficient evidence to support the claim that St. John's Wort is effective in treating major depression.

Uji hipotesis dua variansi populasi Normal

i. Susun hipotesis

$$a.\ H_0 : \sigma_1^2 = \sigma_2^2$$

$$H_1 : \sigma_1^2 \neq \sigma_2^2$$

$$b.\ H_0 : \sigma_1^2 = \sigma_2^2$$

$$H_1 : \sigma_1^2 > \sigma_2^2$$

$$c.\ H_0 : \sigma_1^2 = \sigma_2^2$$

$$H_1 : \sigma_1^2 < \sigma_2^2$$

ii. Pilih tingkat signifikansi α

iii. Statistika uji $F_0 = \frac{S_1^2}{S_2^2}$

iv. Daerah kritis

$a.\ H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$ ⟹ $f_0 > f_{\frac{\alpha}{2}, n_1-1, n_2-1}$ atau $f_0 < f_{1-\frac{\alpha}{2}, n_1-1, n_2-1}$

$b.\ H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 > \sigma_2^2$ ⟹ $f_0 > f_{\alpha, n_1-1, n_2-1}$

$c.\ H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 < \sigma_2^2$ ⟹ $f_0 < f_{1-\alpha, n_1-1, n_2-1}$

contoh

$n_1 = 20, \quad n_2 = 20$

$s_1^2 = 3{,}84 \quad s_2^2 = 4{,}54$

i. $H_0 : \sigma_1^2 = \sigma_2^2$

$H_0 : \sigma_1^2 \neq \sigma_2^2$

ii. $\alpha = 5\%$

iii. $f_0 = \dfrac{s_1^2}{s_2^2} = \dfrac{3{,}84}{4{,}54} = 0{,}85$

iv. Tolak H_0 jika $f_0 > f_{0,025;19;19} = 2{,}53$

atau $f_0 < f_{0,975;19;19} = \dfrac{1}{f_{0,025;19;19}} = 0{,}4$

karena $f_{0,975;19;19} = 0{,}4 < f_0 = 0{,}85 < f_{0,025;19;19} = 2{,}53$

maka H_0 tidak di tolak