Chapter 1_3

- Determinan matriks orde dua dan orde tiga
  - Invers matriks orde dua dan orde tiga

Determinan Matriks

- Determinan matriks **A** dinotasikan dengan **|A|** digunakan dalam statistika multivariat :

1. Determinan matrik kovariansi merepresentasikan variansi untuk beberapa variabel
2. Determinan merupakan representasi variansi untuk suatu kumpulan variabel

Contoh menentukan determinan matriks 2x2

$$\mathbf{A} = \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix} \Rightarrow |\mathbf{A}| = 4 \cdot (2) - 1 \cdot (1) = 7$$

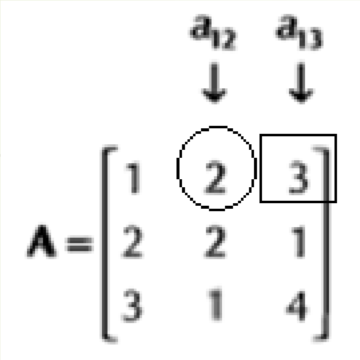
- Secara umum untuk matrik 2x2

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow |\mathbf{A}| = ad - bc.$$

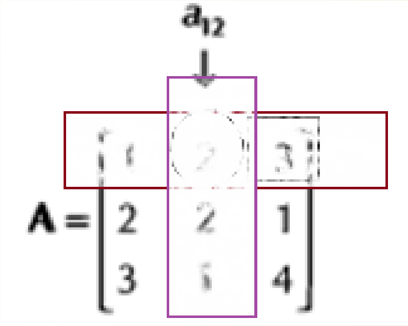
- Untuk menentukan determinan 3x3, kita perlu tahu definisi ini
- definisi

Minor elemen $\mathbf{a}_{ij}$ adalah determinan matrik yang dibentuk dengan menghapus baris ke-i dan kolom ke-j

Contoh menentukan determinan matrik 3x3



$$a_{12} = 2 \Rightarrow \text{minor}\, 2\, \text{adl} \begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} = 8 - 3 = 5$$



$$a_{13} = 3 \Rightarrow \text{minor}\, 3\, \text{adl} \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} = 2 - 6 = -4$$

Definisi kofaktor $a_{ij} = (-1)^{i+j} \text{ x minor}$

- CONTOH

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ 3 & 1 & 4 \end{bmatrix}$$

- STEP 1

$$a_{11} : (-1)^{1+1} = 1$$

$$a_{12} : (-1)^{1+2} = -1$$

$$a_{13} : (-1)^{1+3} = 1$$

STEP 2. PATERN

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

- STEP 3. MATRIK KOFAKTOR

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

DETERMINAN **A**

Determinan diperoleh dengan menghitung baris atau kolom dari matrik kofaktor

$$|A| = a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13}$$

atau

$$|A| = a_{12}c_{12} + a_{22}c_{22} + a_{32}c_{32}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ 3 & 1 & 4 \end{bmatrix}$$

- **misal**

elemen	minor	kofaktor	elemen x kofaktor
$a_{11} = 1$	$\begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix} = 7$	7	7
$a_{12} = 2$	$\begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} = 5$	–5	–10
$a_{13} = 3$	$\begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} = -4$	–4	–12

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

determinan $|A| = 7 + (-10) + (-12) = -15.$

Invers matrik $\mathbf{A}^{-1}$

- **Jika memenuhi**

$$\mathbf{A}\,\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}_n$$

- **In merupakan matrik identitas orde n. Matrik identitas merupakan matrik dengan nilai 1 pada diagonal utama dan 0 selainnya**
- **contoh**

$$\mathbf{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Contoh menentukan matrik 2x2

$$\mathbf{D} = \begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix}$$

Step 1. Cari matrik minor

$$\begin{bmatrix} 6 & 2 \\ 2 & 4 \end{bmatrix}$$

Step 2. patern

$$\begin{bmatrix} + & - \\ - & + \end{bmatrix}$$

Step 3. Cari matrik kofaktor

$$\begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix}$$

- Step 4. Cari determinan

$$D = 6(4) - 2(2) = 20.$$

- Step 5. Cari matriks Invers

$$D^{-1} = \begin{bmatrix} \frac{6}{20} & \frac{-2}{20} \\ \frac{-2}{20} & \frac{4}{20} \end{bmatrix}$$

- *checking*

$$\overset{D}{\begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix}} \overset{D^{-1}}{\begin{bmatrix} \frac{6}{20} & \frac{-2}{20} \\ \frac{-2}{20} & \frac{4}{20} \end{bmatrix}} = \overset{D^{-1}}{\begin{bmatrix} \frac{6}{20} & \frac{-2}{20} \\ \frac{-2}{20} & \frac{4}{20} \end{bmatrix}} \overset{D}{\begin{bmatrix} 4 & 2 \\ 2 & 6 \end{bmatrix}} = \overset{I_2}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

Contoh menentukan invers matriks 3x3

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ 3 & 1 & 4 \end{bmatrix}$$

- Mencari matrik minor

Element	Matrix	Minor
$a_{11} = 1$	$\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$	$2 \times 4 - 1 \times 1 = 7$
$a_{12} = 2$	$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$	$2 \times 4 - 1 \times 3 = 5$
$a_{13} = 3$	$\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$	$2 \times 1 - 2 \times 3 = -4$
$a_{22} = 2$	$\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$	$1 \times 4 - 3 \times 3 = -5$
$a_{23} = 1$	$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$	$1 \times 1 - 2 \times 3 = -5$
$a_{33} = 4$	$\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$	$1 \times 2 - 2 \times 2 = -2$

- Matrik minor

$$\begin{bmatrix} 7 & 5 & -4 \\ 5 & -5 & -5 \\ -4 & -5 & -2 \end{bmatrix}$$

- Patern

$$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix} \Rightarrow \begin{bmatrix} 7 & -5 & -4 \\ -5 & -5 & 5 \\ -4 & 5 & -2 \end{bmatrix}$$

elemen	matriks	minor
$a_{11} = 1$	$\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}$	$2 \times 4 - 1 \times 1 = 7$
$a_{12} = 2$	$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$	$2 \times 4 - 1 \times 3 = 5$
$a_{13} = 3$	$\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$	$2 \times 1 - 2 \times 3 = -4$
$a_{22} = 2$	$\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$	$1 \times 4 - 3 \times 3 = -5$
$a_{23} = 1$	$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$	$1 \times 1 - 2 \times 3 = -5$
$a_{33} = 4$	$\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$	$1 \times 2 - 2 \times 2 = -2$

$$\begin{bmatrix} -7 & -5 & -4 \\ -5 & -5 & 5 \\ -4 & 5 & -2 \end{bmatrix}$$

det **A**=7-10-12=-15

Matriks invers

$$A^{-1} = \begin{bmatrix} \frac{-7}{15} & \frac{1}{3} & \frac{4}{15} \\ \frac{1}{3} & \frac{1}{3} & \frac{-1}{3} \\ \frac{4}{15} & \frac{-1}{3} & \frac{2}{15} \end{bmatrix}$$