Rank Matriks

matrik tak nol A dikatakan mempunyai rank r jika paling sedikit satu dari minor bujur sangkar rxr tidak sama dengan nol sedangkan setiap minor bujur sangkar (r+1)x(r+1), jika ada adalah nol.
Matrik nol mempunyai rank nol

- Merupakan dimensi dari submatrik terbesar **A** yang mempunyai determinan tidak 0

Contoh 1

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix} \qquad |\mathbf{A}| = 0, \text{ misal diambil } \left| \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \right| \neq 0$$

maka A mempunyai rank 2

Contoh 2

$$\begin{bmatrix} 4 & 5 & 2 & 14 \\ 3 & 9 & 6 & 21 \\ 8 & 10 & 7 & 28 \\ 1 & 2 & 9 & 5 \end{bmatrix}$$

Rank matriks = 3

$$det(A) = 0, \text{ tapi } det\left(\begin{bmatrix} 4 & 5 & 2 \\ 3 & 9 & 6 \\ 8 & 10 & 7 \end{bmatrix}\right) = 63 \neq 0$$

Rank matriks

- ***Alternative definition:*** *the maximum number of linearly independent columns (or rows) of **A**.*

$$c_1 v_1 + c_2 v_2 + \cdots + c_k v_k = 0$$

⇓

$$c_1 = c_2 = \cdots = c_k = 0$$

Contoh

$$\mathbf{A} = \begin{bmatrix} 4 & 5 & 2 & 14 \\ 3 & 9 & 6 & 21 \\ 8 & 10 & 7 & 28 \\ 1 & 2 & 9 & 5 \end{bmatrix}$$

$$1\begin{bmatrix} 4 \\ 3 \\ 8 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 5 \\ 9 \\ 10 \\ 2 \end{bmatrix} + 0\begin{bmatrix} 2 \\ 6 \\ 7 \\ 9 \end{bmatrix} - 1\begin{bmatrix} 14 \\ 21 \\ 28 \\ 5 \end{bmatrix} = 0$$

Ranknya bukan 4 !

Theorema : Untuk sembarang matrik, rank baris maupun kolomnya adalah sama

Mencari rank- kolom

cth

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \end{bmatrix}$$

Daftar semua kombinasi kolom

$\Rightarrow$ Rank kolom = 2

Linearly independent??

Y: $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ Y: $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ Y: $\begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$ Y: $\begin{bmatrix} 1 \\ 4 \\ 8 \end{bmatrix}$ Y: $\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 4 \end{bmatrix}$ Y: $\begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}$ Y: $\begin{bmatrix} 1 & 1 \\ 1 & 4 \\ 2 & 8 \end{bmatrix}$ Y: $\begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 4 & 6 \end{bmatrix}$ Y: $\begin{bmatrix} 1 & 1 \\ 2 & 4 \\ 4 & 8 \end{bmatrix}$ Y: $\begin{bmatrix} 1 & 1 \\ 3 & 4 \\ 6 & 8 \end{bmatrix}$

N: $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$ N: $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}$ N: $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 4 \\ 2 & 6 & 8 \end{bmatrix}$ N: $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 4 & 6 & 8 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 6 & 8 \end{bmatrix}$ N

Contoh lain row-rank

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

Linearly independent

$$\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$$

Linearly independent

$$\begin{bmatrix} 2 & 4 & 6 & 8 \end{bmatrix}$$

Linearly independent

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \end{bmatrix}$$

Linearly dependent

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

Linearly independent

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 6 & 8 \end{bmatrix}$$

Linearly independent

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \end{bmatrix}$$

Linearly dependent

$\Rightarrow$ Row-Rank = 2

Matriks Orthogonal

- Notasi:

$$A = \begin{bmatrix} a_{11} & a_{12} & . & a_{1n} \\ a_{21} & a_{22} & . & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & . & a_{mn} \end{bmatrix},$$

$$u_1^T = [a_{11}\ a_{12}\ \cdots\ a_{1n}]$$
$$u_2^T = [a_{21}\ a_{22}\ \cdots\ a_{2n}]$$
$$\cdots$$
$$u_m^T = [a_{m1}\ a_{m2}\ \cdots\ a_{mn}]$$

$$A = \begin{bmatrix} u_1^T \\ u_2^T \\ \cdots \\ u_m^T \end{bmatrix}$$

- A adalah matrik ortogonal jika

$u_j . u_k = 0$, for every $j \neq k$ (u_j is perpendicular to u_k)

contoh

$$\begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}$$