KD 3_2

Two-Way Multivariate Analysis of Variance

An Intro.. Univariat 2 way fixed effect with interaction

- Misal terdapat sebanyak g level faktor 1 dan b level pada faktor 2, serta n observasi independen yang diamati pada setiap kombinasi level-level gb .
- Model :

$$X_{lkr} = \mu + \tau_l + \beta_k + (\tau\beta)_{lk} + e_{lkr}$$

$l = 1,2,...,g$

$k = 1,2,...,b$

$r = 1,2,...,n$

X_{lkr} merupakan observasi ke - r di level l pada faktor 1 dan di level k pada faktor 2

μ merupakan rerata keseluruhan

τ_l merupakan pengaruh level ke - l faktor 1

β_k merupakan pengaruh level ke - k faktor 2

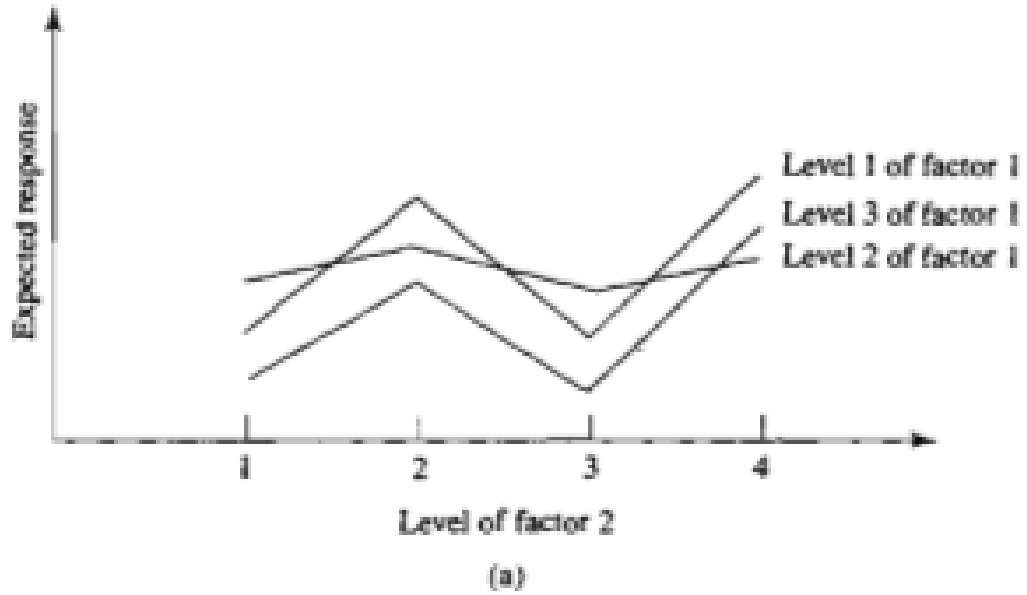
$(\tau\beta)_{lk}$ merupakan pengaruh level ke - l faktor 1 dan level ke - k faktor 2

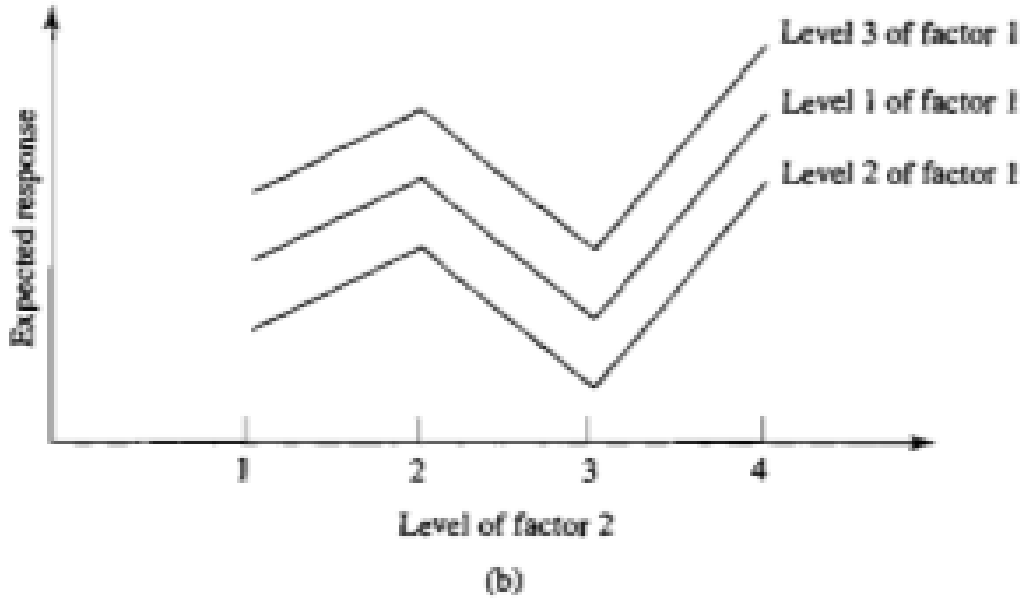
- Dengan asumsi

$$\sum_{l=1}^{g} \tau_l = \sum_{k=1}^{b} \beta_k = \sum_{l=1}^{g} (\tau\beta)_{lk} = \sum_{k=1}^{b} (\tau\beta)_{lk} = 0$$

$$e_{lkr} \sim \text{iid } N(0, \sigma^2)$$

Interaksi...





Curves for expected responses

(a) with interaction

(b) without interaction.

Dekomposisi model...

$$X_{lkr} = \bar{x} + \left(\bar{x}_{l\bullet} - \bar{x}\right) + \left(\bar{x}_{\bullet k} - \bar{x}\right) + \left(\bar{x}_{lk} - \bar{x}_{l} - \bar{x}_{\bullet k} + \bar{x}\right) + \left(x_{lkr} - \bar{x}_{lk}\right)$$

Untuk suatu x

$$x_{lkr} = \bar{x} + \left(\bar{x}_{l\bullet} - \bar{x}\right) + \left(\bar{x}_{\bullet k} - \bar{x}\right) + \left(\bar{x}_{lk} - \bar{x}_{l} - \bar{x}_{\bullet k} + \bar{x}\right) + \left(x_{lkr} - \bar{x}_{lk}\right)$$

$$\underbrace{\sum_{l=1}^{g}\sum_{k=1}^{b}\sum_{r=1}^{n}\left(x_{lkr} - \bar{x}\right)^2}_{JK_{tot(kor)}} = \underbrace{bn\sum_{l=1}^{g}\left(\bar{x}_{l\bullet} - \bar{x}\right)^2}_{JK_{fak1}} + \underbrace{gn\sum_{k=1}^{b}\left(\bar{x}_{\bullet k} - \bar{x}\right)^2}_{JK_{fak2}} + \underbrace{n\sum_{l=1}^{g}\sum_{k=1}^{b}\left(\bar{x}_{lk} - \bar{x}_{l} - \bar{x}_{\bullet k} + \bar{x}\right)^2}_{JK_{Int}} + \underbrace{\sum_{l=1}^{g}\sum_{k=1}^{b}\sum_{r=1}^{n}\left(x_{lkr} - \bar{x}_{lk}\right)^2}_{JK_{res}}$$

Jadi

$$JK_{tot(kor)} = JK_{fak1} + JK_{fak2} + JK_{Int} + JK_{res}$$

- Dengan derajat bebas (db)

$$(gbn-1)=(g-1)+(b-1)+(g-1)(b-1)+gb(n-1)$$

Tabel anava

Univariat 2 way fixed effect with interaction

Sumber variansi	JK	db
Faktor 1	$bn\sum_{l=1}^{g}\left(\bar{x}_{l\bullet}-\bar{x}\right)^2$	g-1
Faktor 2	$gn\sum_{k=1}^{b}\left(\bar{x}_{\bullet k}-\bar{x}\right)^2$	b-1
Interaksi	$n\sum_{l=1}^{g}\sum_{k=1}^{b}\left(\bar{x}_{lk}-\bar{x}_{l}-\bar{x}_{\bullet k}+\bar{x}\right)^2$	(g-1)(b-1)
Residual	$\sum_{l=1}^{g}\sum_{k=1}^{b}\sum_{r=1}^{n}\left(x_{lkr}-\bar{x}_{lk}\right)^2$	gb(n-1)
Total (koreksi)	$\sum_{l=1}^{g}\sum_{k=1}^{b}\sum_{r=1}^{n}\left(x_{lkr}-\bar{x}\right)^2$	gbn-1

Multivariate Two-Way Fixed-Effects Model with Interaction .. (Analog dengan yang univariat)

- Model linier:

$$\mathbf{X}_{lkr} = \boldsymbol{\mu} + \boldsymbol{\tau}_l + \boldsymbol{\beta}_k + \boldsymbol{\gamma}_{lk} + \mathbf{e}_{lkr}$$
$$l = 1,2,\ldots,g$$
$$k = 1,2,\ldots,b$$
$$r = 1,2,\ldots,n$$

$$\mathbf{X}_{lkr} = \bar{\mathbf{x}} + \left(\bar{\mathbf{x}}_{l\bullet} - \bar{\mathbf{x}}\right) + \left(\bar{\mathbf{x}}_{\bullet k} - \bar{\mathbf{x}}\right) + \left(\bar{\mathbf{x}}_{lk} - \bar{\mathbf{x}}_{l} - \bar{\mathbf{x}}_{\bullet k} + \bar{\mathbf{x}}\right) + \left(\mathbf{x}_{lkr} - \bar{\mathbf{x}}_{lk}\right)$$

Rata-rata keseluruhan vektor

Rata-rata observasi vektor pada tingkat ke-l faktor 1

Rata-rata observasi vektor pada tingkat ke-k faktor 2

Rata-rata observasi vektor pada tingkat ke-l faktor 1 dan tingkat ke-k faktor 2

- Dengan asumsi

$$\sum_{l=1}^{g}\boldsymbol{\tau}_l = \sum_{k=1}^{b}\boldsymbol{\beta}_k = \sum_{l=1}^{g}\boldsymbol{\gamma}_{lk} = \sum_{k=1}^{b}\boldsymbol{\gamma}_{lk} = \mathbf{0}$$

$$\mathbf{e}_{lkr} \sim \text{iid}\, N_p(\mathbf{0}, \Sigma)$$

The vectors are all of order px1

The responses consist of p measurements replicated n times at each of the possible combinations of levels of factors 1 and factors 2

Tabel Mulitivariat Manova 2 way

SV	Matrix of sum of Squares and cross products (SSP)	df
Faktor 1	$SSP_{fak1} = \sum_{l=1}^{g} bn\left(\bar{\mathbf{x}}_{l\bullet} - \bar{\mathbf{x}}\right)\left(\bar{\mathbf{x}}_{l\bullet} - \bar{\mathbf{x}}\right)'$	g-1
Faktor 2	$SSP_{fak2} = \sum_{k=1}^{b} gn\left(\bar{\mathbf{x}}_{\bullet k} - \bar{\mathbf{x}}\right)\left(\bar{\mathbf{x}}_{\bullet k} - \bar{\mathbf{x}}\right)'$	b-1
Interaksi	$SSP_{\text{int}} = \sum_{l=1}^{g}\sum_{k=1}^{b} n\left(\bar{\mathbf{x}}_{lk} - \bar{\mathbf{x}}_{l\bullet} - \bar{\mathbf{x}}_{\bullet k} + \bar{\mathbf{x}}\right)\left(\bar{\mathbf{x}}_{lk} - \bar{\mathbf{x}}_{l\bullet} - \bar{\mathbf{x}}_{\bullet k} + \bar{\mathbf{x}}\right)'$	(g-1)(b-1)
Residual	$SSP_{res} = \sum_{l=1}^{g}\sum_{k=1}^{b}\sum_{r=1}^{n}\left(\bar{\mathbf{x}}_{lkr} - \bar{\mathbf{x}}_{lk}\right)\left(\bar{\mathbf{x}}_{lkr} - \bar{\mathbf{x}}_{lk}\right)'$	gb(n-1)
Total (koreksi)	$SSP_{kor} = \sum_{l=1}^{g}\sum_{k=1}^{b}\sum_{r=1}^{n}\left(\bar{\mathbf{x}}_{lkr} - \bar{\mathbf{x}}\right)\left(\bar{\mathbf{x}}_{lkr} - \bar{\mathbf{x}}\right)'$	gbn-1

$$\sum_{\ell=1}^{g}\sum_{k=1}^{b}\sum_{r=1}^{n}(\mathbf{x}_{\ell kr} - \bar{\mathbf{x}})(\mathbf{x}_{\ell kr} - \bar{\mathbf{x}})' = \sum_{\ell=1}^{g} bn(\bar{\mathbf{x}}_{\ell\cdot} - \bar{\mathbf{x}})(\bar{\mathbf{x}}_{\ell\cdot} - \bar{\mathbf{x}})'$$
$$+ \sum_{k=1}^{b} gn(\bar{\mathbf{x}}_{\cdot k} - \bar{\mathbf{x}})(\bar{\mathbf{x}}_{\cdot k} - \bar{\mathbf{x}})'$$
$$+ \sum_{\ell=1}^{g}\sum_{k=1}^{b} n(\bar{\mathbf{x}}_{\ell k} - \bar{\mathbf{x}}_{\ell\cdot} - \bar{\mathbf{x}}_{\cdot k} + \bar{\mathbf{x}})(\bar{\mathbf{x}}_{\ell k} - \bar{\mathbf{x}}_{\ell\cdot} - \bar{\mathbf{x}}_{\cdot k} + \bar{\mathbf{x}})'$$
$$+ \sum_{\ell=1}^{g}\sum_{k=1}^{b}\sum_{r=1}^{n}(\mathbf{x}_{\ell kr} - \bar{\mathbf{x}}_{\ell k})(\mathbf{x}_{\ell kr} - \bar{\mathbf{x}}_{\ell k})'$$

db:

$gbn - 1 = (g - 1) + (b - 1) + (g - 1)(b - 1) + gb(n - 1)$

The Likelihood Ratio Test

Biasanya, test untuk interaksi dilakukan sebelum test untuk efek dari faktor yang utama.
Bila terdapat efek dari interaksi maka efek-efek dari faktor tidak memiliki penafsiran yang jelas.
Hal ini mengakibatkan tidak baik untuk melakukan test multivariate selanjutnya.
Uji likelihood dilakukan jika

$$p \le gb(n-1)$$

i. Susun Hipotesis

$$H_{0AB} : \gamma_{11} = \gamma_{12} = \cdots = \gamma_{gb} = \mathbf{0}$$

$$H_{1AB} : \text{paling tidak satu } \gamma_{lk} \neq \mathbf{0}$$

ii. Pilih tingkat signifikansi

iii. Hitungan

$$\Lambda^* = \frac{|SSP_{\text{res}}|}{|SSP_{\text{int}} + SSP_{\text{res}}|}$$

Tolak H0 jika $$-\left[gb(n-1) - \frac{p+1-(g-1)(b-1)}{2}\right] \ln \Lambda^* > \chi^2_{(g-1)(b-1)p}(\alpha)$$

atau

$$F = \left(\frac{1-\Lambda^*}{\Lambda^*}\right) \frac{(gb(n-1)-p+1)/2}{(|(g-1)(b-1)-p|+1)/2} > F_{\alpha,\nu_1,\nu_2}$$

$$\nu_1 = |(g-1)(b-1)-p|+1$$

$$\nu_2 = gb(n-1)-p+1$$

Uji Efek Faktor 1

i. Susun Hipotesis

$$H_{0A} : \boldsymbol{\tau}_1 = \boldsymbol{\tau}_2 = \cdots = \boldsymbol{\tau}_g = \mathbf{0}$$

$$H_{1A} : \text{paling tidak satu } \boldsymbol{\tau}_l \neq \mathbf{0}$$

ii. Pilih tingkat signifikansi

iii. Statistika Uji

$$\Lambda_1^* = \frac{\left|SSP_{\text{res}}\right|}{\left|SSP_{\text{fac}1} + SSP_{\text{res}}\right|}$$

Tolak H0 jika

$$-\left[gb(n-1) - \frac{p+1-(g-1)}{2}\right] \ln \Lambda_1^* > \chi^2_{(g-1)p}(\alpha)$$

atau

$$F_1 = \left(\frac{1-\Lambda_1^*}{\Lambda_1^*}\right) \frac{(gb(n-1)-p+1)/2}{(|(g-1)-p|+1)/2} > F_{\alpha,\nu_1,\nu_2}$$

$$\nu_1 = |(g-1)-p|+1$$

$$\nu_2 = gb(n-1)-p+1$$

Uji Efek Faktor 2

i. Susun Hipotesis

$$H_{0B}: \boldsymbol{\beta}_1 = \boldsymbol{\beta}_2 = \cdots = \boldsymbol{\beta}_b = \mathbf{0}$$

$$H_{1B}: \text{paling tidak satu } \boldsymbol{\beta}_k \neq \mathbf{0}$$

ii. Pilih tingkat signifikansi

iii. Statistika Uji

$$\Lambda_2^{\,*} = \frac{\left|SSP_{\text{res}}\right|}{\left|SSP_{\text{fac}2} + SSP_{\text{res}}\right|}$$

Tolak H0 jika

$$-\left[gb(n-1) - \frac{p+1-(b-1)}{2}\right] \ln \Lambda_2^{\,*} > \chi^2_{(b-1)p}(\alpha)$$

atau

$$F_2 = \left(\frac{1-\Lambda_2^{\,*}}{\Lambda_2^{\,*}}\right) \frac{\left(gb(n-1) - p + 1\right)/2}{\left(\left|(b-1) - p\right| + 1\right)/2} > F_{\alpha,\nu_1,\nu_2}$$

$$\nu_1 = \left|(b-1) - p\right| + 1$$

$$\nu_2 = gb(n-1) - p + 1$$

contoh

Kondisi yang optimum untuk meng-extruding film plastik digunakan suatu teknik, yang disebut *Evolutionary Operation*. Terdapat 3 variabel yang diukur yaitu x1=*tear resistance*, x2=*gloss*, x3=*opacity*. Pengukuran dilakukan pada dua level di setiap faktor, *rate of extrusion* dan *ammount of additive*. Pengukuran dilakukan sebanyak n=5 kali pada setiap kombinasi pada level setiap faktor. Hasil pengukuran terdapat pada tabel di samping.

x_1=*tear resistance*, x_2=*gloss*, x_3=*opacity*

		Factor 2: amount of additive					
		Low (1.0%)			High (1.5%)		
Factor 1:change in rate of extrusion	Low (-10%)	x_1	x_2	x_3	x_1	x_2	x_3
		6.5	9.5	4.4	6.9	9.1	5.7
		6.2	9.9	6.4	7.2	10	2.0
		5.8	9.6	3.0	6.9	9.9	3.9
		6.5	9.6	4.1	6.1	9.5	1.9
		6.5	9.2	0.8	6.3	9.4	5.7
	High (10%)	x_1	x_2	x_3	x_1	x_2	x_3
		6.7	9.1	2.8	7.1	9.2	8.4
		6.6	9.3	4.1	7.0	8.8	5.2
		7.2	8.3	3.8	7.2	9.7	6.9
		7.1	8.4	1.6	7.5	10.1	2.7
		6.8	8.5	3.4	7.6	9.2	1.9

penyelesaian

Diketahui g=2, b=2, n=5, p=3

x_1=*tear resistance*, x_2=*gloss*, x_3=*opacity*

		Factor 2: amount of additive					
		Low (1.0%)			High (1.5%)		
		x_1	x_2	x_3	x_1	x_2	x_3
Factor 1:change in rate of extrusion	Low (-10%)	6.5	9.5	4.4	6.9	9.1	5.7
		6.2	9.9	6.4	7.2	10	2.0
		5.8	9.6	3.0	6.9	9.9	3.9
		6.5	9.6	4.1	6.1	9.5	1.9
		6.5	9.2	0.8	6.3	9.4	5.7
	High (10%)	x_1	x_2	x_3	x_1	x_2	x_3
		6.7	9.1	2.8	7.1	9.2	8.4
		6.6	9.3	4.1	7.0	8.8	5.2
		7.2	8.3	3.8	7.2	9.7	6.9
		7.1	8.4	1.6	7.5	10.1	2.7
		6.8	8.5	3.4	7.6	9.2	1.9

$$SSP_{fac1} = \sum_{l=1}^{g} bn(\bar{x}_{l.} - \bar{x})(\bar{x}_{l.} - \bar{x})'$$

$$\bar{x}_{2.} = \begin{bmatrix} 7,08 \\ 9,06 \\ 4,08 \end{bmatrix} \quad \bar{x}_{1.} = \begin{bmatrix} 6,49 \\ 9,57 \\ 3,79 \end{bmatrix} \quad \bar{x} = \begin{bmatrix} 6,785 \\ 9,315 \\ 3,935 \end{bmatrix}$$

$$\bar{x}_{1.} - \bar{x} = \begin{bmatrix} 6,49 \\ 9,57 \\ 3,79 \end{bmatrix} - \begin{bmatrix} 6,785 \\ 9,315 \\ 3,935 \end{bmatrix} = \begin{bmatrix} -0,295 \\ 0,255 \\ -0,145 \end{bmatrix}$$

$$(\bar{x}_{1.} - \bar{x})(\bar{x}_{1.} - \bar{x})' = \begin{bmatrix} 0,087025 & -0,075225 & 0,042775 \\ & 0,065025 & -0,036975 \\ & & 0,021025 \end{bmatrix}$$

$$\bar{x}_{2.} - \bar{x} = \begin{bmatrix} 7,08 \\ 9,06 \\ 4,08 \end{bmatrix} - \begin{bmatrix} 6,785 \\ 9,315 \\ 3,935 \end{bmatrix} = \begin{bmatrix} 0,295 \\ -0,255 \\ 0,145 \end{bmatrix}$$

$$\mathbf{X}_{lkr} = \boldsymbol{\mu} + \boldsymbol{\tau}_l + \boldsymbol{\beta}_k + \boldsymbol{\gamma}_{lk} + \mathbf{e}_{lkr}$$

$$l = 1,2,\ldots,g$$

$$k = 1,2,\ldots,b$$

$$r = 1,2,\ldots,n$$

x_1=tear resistance, x_2=gloss, x_3=opacity

		Factor 2: amount of additive					
		Low (1.0%)			High (1.5%)		
Factor 1:change in rate of extrusion	Low (-10%)	x_1	x_2	x_3	x_1	x_2	x_3
		6.5	9.5	4.4	6.9	9.1	5.7
		6.2	9.9	6.4	7.2	10	2.0
		5.8	9.6	3.0	6.9	9.9	3.9
		6.5	9.6	4.1	6.1	9.5	1.9
		6.5	9.2	0.8	6.3	9.4	5.7
	High (10%)	x_1	x_2	x_3	x_1	x_2	x_3
		6.7	9.1	2.8	7.1	9.2	8.4
		6.6	9.3	4.1	7.0	8.8	5.2
		7.2	8.3	3.8	7.2	9.7	6.9
		7.1	8.4	1.6	7.5	10.1	2.7
		6.8	8.5	3.4	7.6	9.2	1.9

$$SSP_{fac2} = \sum_{k=1}^{b} gn(\bar{x}_{.k} - \bar{x})(\bar{x}_{.k} - \bar{x})'$$

$$\bar{x}_{.1} = \begin{bmatrix} 6,59 \\ 9,14 \\ 3,44 \end{bmatrix} \qquad \bar{x}_{.2} = \begin{bmatrix} 6,98 \\ 9,49 \\ 4,43 \end{bmatrix} \qquad \bar{x} = \begin{bmatrix} 6,785 \\ 9,315 \\ 3,935 \end{bmatrix}$$

$$\bar{x}_{.1} - \bar{x} = \begin{bmatrix} 6,59 \\ 9,14 \\ 3,44 \end{bmatrix} - \begin{bmatrix} 6,785 \\ 9,315 \\ 3,935 \end{bmatrix} = \begin{bmatrix} -0,195 \\ -0,175 \\ -0,495 \end{bmatrix}$$

$$(\bar{x}_{.1} - \bar{x})(\bar{x}_{.1} - \bar{x})' = \begin{bmatrix} 0,038025 & 0,034125 & 0,096525 \\ & 0,030625 & 0,086625 \\ & & 0,245025 \end{bmatrix}$$

$$\bar{x}_{.2} - \bar{x} = \begin{bmatrix} 6,98 \\ 9,49 \\ 4,43 \end{bmatrix} - \begin{bmatrix} 6,785 \\ 9,315 \\ 3,935 \end{bmatrix} = \begin{bmatrix} 0,195 \\ 0,175 \\ 0,495 \end{bmatrix}$$

$$(\bar{x}_{.2} - \bar{x})(\bar{x}_{.2} - \bar{x})' = \begin{bmatrix} 0,038025 & 0,034125 & 0,096525 \\ & 0,030625 & 0,086625 \\ & & 0,245025 \end{bmatrix}$$

$$SSP_{fac2} = (2 \times 5)\left(2 \begin{bmatrix} 0,038025 & 0,034125 & 0,096525 \\ & 0,030625 & 0,086625 \\ & & 0,245025 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0,7605 & 0,6825 & 1,9305 \\ & 0,6125 & 1,7325 \\ & & 4,9005 \end{bmatrix}$$

x_1=tear resistance, x_2=gloss, x_3=opacity

		Factor 2: amount of additive					
		Low (1.0%)			High (1.5%)		
Factor 1:change in rate of extrusion	Low (-10%)	x_1	x_2	x_3	x_1	x_2	x_3
		6.5	9.5	4.4	6.9	9.1	5.7
		6.2	9.9	6.4	7.2	10	2.0
		5.8	9.6	3.0	6.9	9.9	3.9
		6.5	9.6	4.1	6.1	9.5	1.9
		6.5	9.2	0.8	6.3	9.4	5.7
	High (10%)	x_1	x_2	x_3	x_1	x_2	x_3
		6.7	9.1	2.8	7.1	9.2	8.4
		6.6	9.3	4.1	7.0	8.8	5.2
		7.2	8.3	3.8	7.2	9.7	6.9
		7.1	8.4	1.6	7.5	10.1	2.7
		6.8	8.5	3.4	7.6	9.2	1.9

$$SSP_{\text{int}} = \sum_{l=1}^{g}\sum_{k=1}^{b} n(\bar{x}_{lk} - \bar{x}_{l.} - \bar{x}_{.k} + \bar{x})(\bar{x}_{lk} - \bar{x}_{l.} - \bar{x}_{.k} + \bar{x})'$$

$$\bar{x}_{11} = \begin{bmatrix} 6,3 \\ 9,56 \\ 3,74 \end{bmatrix} \quad \bar{x}_{12} = \begin{bmatrix} 6,68 \\ 9,58 \\ 3,84 \end{bmatrix} \quad \bar{x}_{21} = \begin{bmatrix} 6,88 \\ 8,72 \\ 3,14 \end{bmatrix} \quad \bar{x}_{22} = \begin{bmatrix} 7,28 \\ 9,4 \\ 3,02 \end{bmatrix}$$

$$\bar{x}_{11} - \bar{x}_{1.} - \bar{x}_{.1} + \bar{x} = \begin{bmatrix} 6,3 \\ 9,56 \\ 3,74 \end{bmatrix} - \begin{bmatrix} 6,49 \\ 9,57 \\ 3,79 \end{bmatrix} - \begin{bmatrix} 6,59 \\ 9,14 \\ 3,44 \end{bmatrix} + \begin{bmatrix} 6,785 \\ 9,315 \\ 3,935 \end{bmatrix} = \begin{bmatrix} 0,005 \\ 0,165 \\ 0,445 \end{bmatrix}$$

$$(\bar{x}_{11} - \bar{x}_{1.} - \bar{x}_{.1} + \bar{x})(\bar{x}_{11} - \bar{x}_{1.} - \bar{x}_{.1} + \bar{x})' = \begin{bmatrix} 0,000025 & 0,000825 & 0,002225 \\ & 0,027225 & 0,073425 \\ & & 0,198025 \end{bmatrix}$$

$$\bar{x}_{12} - \bar{x}_{1.} - \bar{x}_{.2} + \bar{x} = \begin{bmatrix} 6{,}68 \\ 9{,}58 \\ 3{,}84 \end{bmatrix} - \begin{bmatrix} 6{,}49 \\ 9{,}57 \\ 3{,}79 \end{bmatrix} - \begin{bmatrix} 6{,}98 \\ 9{,}49 \\ 4{,}43 \end{bmatrix} + \begin{bmatrix} 6{,}785 \\ 9{,}315 \\ 3{,}935 \end{bmatrix} = \begin{bmatrix} -0{,}005 \\ -0{,}165 \\ -0{,}445 \end{bmatrix}$$

$$(\bar{x}_{12} - \bar{x}_{1.} - \bar{x}_{.2} + \bar{x})(\bar{x}_{12} - \bar{x}_{1.} - \bar{x}_{.2} + \bar{x})' = \begin{bmatrix} 0{,}000025 & 0{,}000825 & 0{,}002225 \\ & 0{,}027225 & 0{,}073425 \\ & & 0{,}198025 \end{bmatrix}$$

$$\bar{x}_{22} - \bar{x}_{2.} - \bar{x}_{.2} + \bar{x} = \begin{bmatrix} 7{,}28 \\ 9{,}4 \\ 5{,}02 \end{bmatrix} - \begin{bmatrix} 7{,}08 \\ 9{,}06 \\ 4{,}08 \end{bmatrix} - \begin{bmatrix} 6{,}98 \\ 9{,}49 \\ 4{,}43 \end{bmatrix} + \begin{bmatrix} 6{,}785 \\ 9{,}315 \\ 3{,}935 \end{bmatrix} = \begin{bmatrix} 0{,}005 \\ 0{,}165 \\ 0{,}445 \end{bmatrix}$$

$$(\bar{x}_{22} - \bar{x}_{2.} - \bar{x}_{.2} + \bar{x})(\bar{x}_{22} - \bar{x}_{2.} - \bar{x}_{.2} + \bar{x})' = \begin{bmatrix} 0{,}000025 & 0{,}000825 & 0{,}002225 \\ & 0{,}027225 & 0{,}073425 \\ & & 0{,}198025 \end{bmatrix}$$

Diperoleh:

$$\begin{aligned} (\bar{x}_{11} - \bar{x}_{1.} - \bar{x}_{.1} + \bar{x})(\bar{x}_{11} - \bar{x}_{1.} - \bar{x}_{.1} + \bar{x})' &= (\bar{x}_{12} - \bar{x}_{1.} - \bar{x}_{.2} + \bar{x})(\bar{x}_{12} - \bar{x}_{1.} - \bar{x}_{.2} + \bar{x})' \\ &= (\bar{x}_{21} - \bar{x}_{2.} - \bar{x}_{.1} + \bar{x})(\bar{x}_{21} - \bar{x}_{2.} - \bar{x}_{.1} + \bar{x})' \\ &= (\bar{x}_{22} - \bar{x}_{2.} - \bar{x}_{.2} + \bar{x})(\bar{x}_{22} - \bar{x}_{2.} - \bar{x}_{.2} + \bar{x})' \end{aligned}$$

$$SSP_{\text{int}} = 5\left(4\begin{bmatrix} 0{,}000025 & 0{,}000825 & 0{,}002225 \\ & 0{,}027225 & 0{,}073425 \\ & & 0{,}198025 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0{,}0005 & 0{,}0165 & 0{,}0445 \\ & 0{,}5445 & 1{,}4685 \\ & & 3{,}9605 \end{bmatrix}$$

$$SSP_{res} = \sum_{l=1}^{g}\sum_{k=1}^{b}\sum_{r=1}^{n}(x_{lkr} - \bar{x}_{lk})(x_{lkr} - \bar{x}_{lk})'$$

$$x_{111} = \begin{bmatrix} 6,5 \\ 9,5 \\ 4,4 \end{bmatrix} \quad x_{112} = \begin{bmatrix} 6,2 \\ 9,9 \\ 6,4 \end{bmatrix} \quad x_{113} = \begin{bmatrix} 5,8 \\ 9,6 \\ 3 \end{bmatrix} \quad x_{114} = \begin{bmatrix} 6,5 \\ 9,6 \\ 4,1 \end{bmatrix} \quad x_{115} = \begin{bmatrix} 6,5 \\ 9,2 \\ 0,8 \end{bmatrix}$$

$$x_{121} = \begin{bmatrix} 6,9 \\ 9,1 \\ 5,7 \end{bmatrix} \quad x_{122} = \begin{bmatrix} 7,2 \\ 10 \\ 2 \end{bmatrix} \quad x_{123} = \begin{bmatrix} 6,9 \\ 9,9 \\ 3,9 \end{bmatrix} \quad x_{124} = \begin{bmatrix} 6,1 \\ 9,5 \\ 1,9 \end{bmatrix} \quad x_{125} = \begin{bmatrix} 6,3 \\ 9,4 \\ 5,7 \end{bmatrix}$$

$$x_{211} = \begin{bmatrix} 6,7 \\ 9,1 \\ 2,8 \end{bmatrix} \quad x_{212} = \begin{bmatrix} 6,6 \\ 9,3 \\ 4,1 \end{bmatrix} \quad x_{213} = \begin{bmatrix} 7,2 \\ 8,3 \\ 3,8 \end{bmatrix} \quad x_{214} = \begin{bmatrix} 7,1 \\ 8,4 \\ 1,6 \end{bmatrix} \quad x_{215} = \begin{bmatrix} 6,8 \\ 8,5 \\ 3,4 \end{bmatrix}$$

$$x_{221} = \begin{bmatrix} 7,1 \\ 9,2 \\ 8,4 \end{bmatrix} \quad x_{222} = \begin{bmatrix} 7 \\ 8,8 \\ 5,2 \end{bmatrix} \quad x_{223} = \begin{bmatrix} 7,2 \\ 9,7 \\ 6,9 \end{bmatrix} \quad x_{224} = \begin{bmatrix} 7,5 \\ 10,1 \\ 2,7 \end{bmatrix} \quad x_{225} = \begin{bmatrix} 7,6 \\ 9,2 \\ 1,9 \end{bmatrix}$$

$$x_{111} - \bar{x}_{11} = \begin{bmatrix} 6,5 \\ 9,5 \\ 44 \end{bmatrix} - \begin{bmatrix} 6,3 \\ 9,56 \\ 3,74 \end{bmatrix} = \begin{bmatrix} 0,2 \\ -0,06 \\ 0,66 \end{bmatrix}$$

x_1=tear resistance, x_2=gloss, x_3=opacity

		Factor 2: amount of additive					
		Low (1.0%)			High (1.5%)		
		x_1	x_2	x_3	x_1	x_2	x_3
Factor 1:change in rate of extrusion	Low (-10%)	6.5	9.5	4.4	6.9	9.1	5.7
		6.2	9.9	6.4	7.2	10	2.0
		5.8	9.6	3.0	6.9	9.9	3.9
		6.5	9.6	4.1	6.1	9.5	1.9
		6.5	9.2	0.8	6.3	9.4	5.7
		x_1	x_2	x_3	x_1	x_2	x_3
	High (10%)	6.7	9.1	2.8	7.1	9.2	8.4
		6.6	9.3	4.1	7.0	8.8	5.2
		7.2	8.3	3.8	7.2	9.7	6.9
		7.1	8.4	1.6	7.5	10.1	2.7
		6.8	8.5	3.4	7.6	9.2	1.9

$$(x_{111} - \bar{x}_{11})(x_{111} - \bar{x}_{11})' = \begin{bmatrix} 0.04 & -0,012 & 0,132 \\ & 0,0036 & -0,0396 \\ & & 0,4356 \end{bmatrix}$$

Dengan menggunakan rumus $(x_{lkr} - \bar{x}_{lk})(x_{lkr} - \bar{x}_{lk})'$ pada semua Xlkr maka

$$SSP_{res} = \begin{bmatrix} 1,7640 & 0,0200 & -3,0700 \\ & 2,2680 & -0,5520 \\ & & 64,9240 \end{bmatrix}$$

sehingga

$$SSP_{cor} = SSP_{fac1} + SSP_{fac2} + SSP_{int} + SSP_{res}$$

$$= \begin{bmatrix} 4,2655 & -0,7855 & -0,2395 \\ & 5,0855 & 1,9095 \\ & & 74,2055 \end{bmatrix}$$

Tabel Manova

Sumber Variasi	SSP	dk
Factor 1: Change in rate of extrusion	$\begin{pmatrix} 1{,}7405 & -1{,}5045 & 0{,}8555 \\ & 1{,}3005 & -0{,}7395 \\ & & 0{,}4205 \end{pmatrix}$	1
Factor 2: Amount of additive	$\begin{pmatrix} 0{,}7605 & 0{,}6825 & 1{,}9305 \\ & 0{,}6125 & 1{,}7325 \\ & & 4{,}9005 \end{pmatrix}$	1
Interaction	$\begin{pmatrix} 0{,}0005 & 0{,}0165 & 0{,}0445 \\ & 0{,}5445 & 1{,}4685 \\ & & 3{,}9605 \end{pmatrix}$	1
Residual	$\begin{pmatrix} 1{,}7640 & 0{,}0200 & -3{,}0700 \\ & 2{,}2680 & -0{,}5520 \\ & & 64{,}9240 \end{pmatrix}$	16
Total (corrected)	$\begin{pmatrix} 4{,}2655 & -0{,}7855 & -0{,}2395 \\ & 5{,}0855 & 1{,}9095 \\ & & 74{,}2055 \end{pmatrix}$	19

Uji untuk Interaksi

i. Susun Hipotesis

$$H_{0AB} : \gamma_{11} = \gamma_{12} = \cdots = \gamma_{gb} = \mathbf{0}$$

$$H_{1AB} : \text{paling tidak satu } \gamma_{lk} \neq \mathbf{0}$$

ii. Pilih tingkat signifikansi

iii. Hitungan

$$\Lambda^* = \frac{|SSP_{res}|}{|SSP_{int} + SSP_{res}|} = \frac{275{,}7098}{354{,}7906} = 0{,}7771$$

F hitung:

$$F = \left(\frac{1-0{,}7771}{0{,}7771}\right)\frac{\left(2(2)(5-1)-3+1\right)/2}{\left(|(2-1)(2-1)-3|+1\right)/2} = 1{,}34$$

Sumber Variasi	SSP	dk
Factor 1: Change in rate of extrusion	$\begin{pmatrix} 1{,}7405 & -1{,}5045 & 0{,}8555 \\ & 1{,}3005 & -0{,}7395 \\ & & 0{,}4205 \end{pmatrix}$	1
Factor 2: Amount of additive	$\begin{pmatrix} 0{,}7605 & 0{,}6825 & 1{,}9305 \\ & 0{,}6125 & 1{,}7325 \\ & & 4{,}9005 \end{pmatrix}$	1
Interaction	$\begin{pmatrix} 0{,}0005 & 0{,}0165 & 0{,}0445 \\ & 0{,}5445 & 1{,}4685 \\ & & 3{,}9605 \end{pmatrix}$	1
Residual	$\begin{pmatrix} 1{,}7640 & 0{,}0200 & -3{,}0700 \\ & 2{,}2680 & -0{,}5520 \\ & & 64{,}9240 \end{pmatrix}$	16
Total (corrected)	$\begin{pmatrix} 4{,}2655 & -0{,}7855 & -0{,}2395 \\ & 5{,}0855 & 1{,}9095 \\ & & 74{,}2055 \end{pmatrix}$	19

$$\nu_1 = |(g-1)(b-1) - p| + 1 = |(2-1)(2-1) - 3| + 1 = 3$$

$$\nu_2 = gb(n-1) - p + 1 = 2(2)(5-1) - 3 + 1 = 14$$

$$F_{\text{tabel}} = F_{0{,}05;\nu_1,\nu_2} = F_{0{,}05;3,14} = 3{,}34$$

Karena $F = 1{,}34 < F_{0{,}05;3,14} = 3{,}34$ maka H_{0AB} diterima

Kesimpulan :
Tidak ada pengaruh yang signifikan antara Faktor 1 (*change in rate of extrusion*) dan Faktor 2 (*amount of additive*) dalam proses extruding

Uji Efek Faktor 1

i. Susun Hipotesis

$H_{0A}: \boldsymbol{\tau}_1 = \boldsymbol{\tau}_2 = \cdots = \boldsymbol{\tau}_g = \mathbf{0}$

$H_{1A}: \text{paling tidak satu } \boldsymbol{\tau}_l \neq \mathbf{0}$

ii. Pilih tingkat signifikansi

iii. Statistika Uji

$$\Lambda_1^* = \frac{|SSP_{\text{res}}|}{|SSP_{\text{fac1}} + SSP_{\text{res}}|} = \frac{275{,}7098}{722{,}0212} = 0{,}3819$$

Tolak H0 jika

$$-\left[gb(n-1) - \frac{p+1-(g-1)}{2}\right] \ln \Lambda_1^* > \chi^2_{(g-1)p}(\alpha)$$

atau

$$F_1 = \left(\frac{1-\Lambda_1^*}{\Lambda_1^*}\right)\frac{(gb(n-1)-p+1)/2}{(|(g-1)-p|+1)/2}$$

$$= \left(\frac{1-0{,}3819}{0{,}3819}\right)\frac{(16-3+1)/2}{(|1-3|+1)/2} = 7{,}55$$

Sumber Variasi	SSP	dk
Factor 1: Change in rate of extrusion	$\begin{pmatrix} 1{,}7405 & -1{,}5045 & 0{,}8555 \\ & 1{,}3005 & -0{,}7395 \\ & & 0{,}4205 \end{pmatrix}$	1
Factor 2: Amount of additive	$\begin{pmatrix} 0{,}7605 & 0{,}6825 & 1{,}9305 \\ & 0{,}6125 & 1{,}7325 \\ & & 4{,}9005 \end{pmatrix}$	1
Interaction	$\begin{pmatrix} 0{,}0005 & 0{,}0165 & 0{,}0445 \\ & 0{,}5445 & 1{,}4685 \\ & & 3{,}9605 \end{pmatrix}$	1
Residual	$\begin{pmatrix} 1{,}7640 & 0{,}0200 & -3{,}0700 \\ & 2{,}2680 & -0{,}5520 \\ & & 64{,}9240 \end{pmatrix}$	16
Total (corrected)	$\begin{pmatrix} 4{,}2655 & -0{,}7855 & -0{,}2395 \\ & 5{,}0855 & 1{,}9095 \\ & & 74{,}2055 \end{pmatrix}$	19

$$\nu_1 = |1-3|+1 = 3$$

$$\nu_2 = 16-3+1 = 14$$

$$F_{3,14}(0{,}05) = 3{,}34$$

$$F_1 = 7{,}55 > F_{3,14}(0{,}05) = 3{,}34 \text{ maka } H_{0A} \text{ ditolak}$$

Uji Efek Faktor 2

i. Susun Hipotesis

$H_{0B}: \boldsymbol{\beta}_1 = \boldsymbol{\beta}_2 = \cdots = \boldsymbol{\beta}_b = \mathbf{0}$

$H_{1B}:$ paling tidak satu $\boldsymbol{\beta}_k \neq \mathbf{0}$

ii. Pilih tingkat signifikansi 5%

iii. Statistika Uji

$$\Lambda_2^* = \frac{|SSP_{\text{res}}|}{|SSP_{\text{fac2}} + SSP_{\text{res}}|} = \frac{275{,}7098}{527{,}1347} = 0{,}523$$

Tolak H0 jika

$$-\left[gb(n-1) - \frac{p+1-(b-1)}{2}\right] \ln \Lambda_2^* > \chi^2_{(b-1)p}(\alpha)$$

Sumber Variasi	SSP	dk
Factor 1: Change in rate of extrusion	$\begin{pmatrix} 1{,}7405 & -1{,}5045 & 0{,}8555 \\ & 1{,}3005 & -0{,}7395 \\ & & 0{,}4205 \end{pmatrix}$	1
Factor 2: Amount of additive	$\begin{pmatrix} 0{,}7605 & 0{,}6825 & 1{,}9305 \\ & 0{,}6125 & 1{,}7325 \\ & & 4{,}9005 \end{pmatrix}$	1
Interaction	$\begin{pmatrix} 0{,}0005 & 0{,}0165 & 0{,}0445 \\ & 0{,}5445 & 1{,}4685 \\ & & 3{,}9605 \end{pmatrix}$	1
Residual	$\begin{pmatrix} 1{,}7640 & 0{,}0200 & -3{,}0700 \\ & 2{,}2680 & -0{,}5520 \\ & & 64{,}9240 \end{pmatrix}$	16
Total (corrected)	$\begin{pmatrix} 4{,}2655 & -0{,}7855 & -0{,}2395 \\ & 5{,}0855 & 1{,}9095 \\ & & 74{,}2055 \end{pmatrix}$	19

atau

$$F_2 = \left(\frac{1-\Lambda_2^*}{\Lambda_2^*}\right)\frac{(gb(n-1)-p+1)/2}{(|(b-1)-p|+1)/2} = \left(\frac{1-0{,}523}{0{,}523}\right)\frac{(16-3+1)/2}{(|1-3|+1)/2} = 4{,}26$$

$$F_{3,14}(0{,}05) = 3{,}34$$

$F_2 = 4{,}26 > F_{3,14}(0{,}05) = 3{,}34$ maka H_{0B} ditolak

PR/ TUGAS INDIVIDUAL KD 3
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DIBUKU :
RICHARD A. JOHNSON & DEAN W. WICHERN. 2007. APPLIED MULTIVARIATE STATISTICAL ANALYSIS. 6 ED. PERSON AND PRENTICE HALL. HALAMAN 338