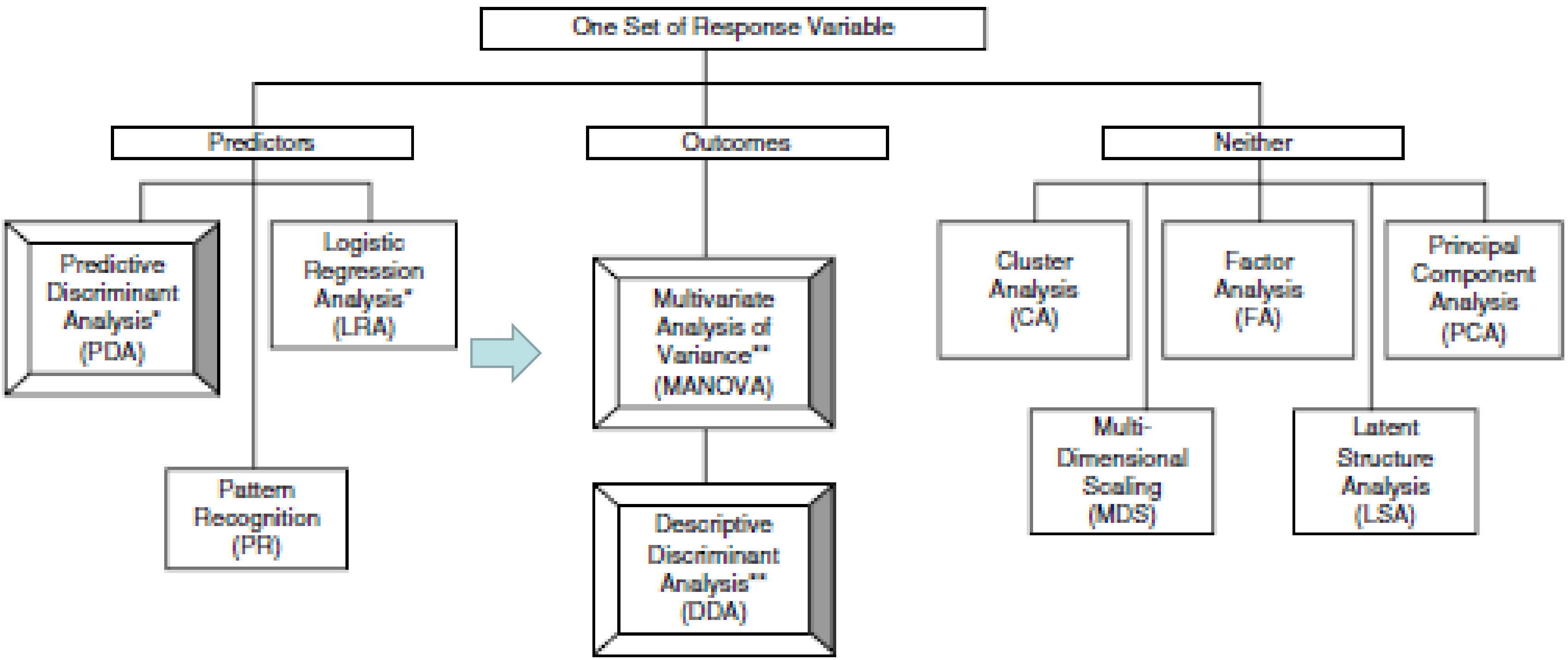
KD 3

One-Way MANOVA

Comparison of Several Multivariate Population Means

CLASSIFICATION OF MULTIVARIATE ANALYSIS (HUBERTY ET AL 2006)



*One grouping variable
**One or more grouping variables

An Intro..

Kadang pada lebih dari dua populasi diambil sampel acak, berupa kumpulan dari tiap g populasi, sebagai berikut

Populasi 1: $X_{11}, X_{12}, \ldots, X_{1n_1}$

Populasi 2: $X_{21}, X_{22}, \ldots, X_{2n_2}$

$\vdots \qquad \vdots$

Populasi g: $X_{g1}, X_{g2}, \ldots, X_{gn_g}$

- MANOVA digunakan untuk meneliti apakah vektor rata-rata populasi itu sama atau tidak,
- jika tidak komponen rata-rata yang mana yang berbeda secara signifikan

Asumsi Struktur Data untuk One-way MANOVA

- X_{i1}, X_{i2},...,X_{in} adalah sampel acak berukuran n_i dari sebuah populasi dengan rata-rata μ_l ,l=1,2,...,g
- Sampel acak dari populasi yang beda adalah independen
- Semua populasi memiliki matrik kovarian bersama Σ
- Tiap populasi adalah normal multivariat

Ingat kembali ttg Univariat ANAVA

- Dalam situasi univariat, asumsi bahwa $X_{l1}, X_{l2}, \ldots, X_{ln}$ adalah sampel acak dari populasi yang berdistribusi $N(\mu_l, \sigma^2)$, l=1,..,g
- Hipotesis :

$$H_0: \mu_l = \mu_2 = \ldots = \mu_g$$

Misal $\mu_l = \mu + (\mu_l - \mu)$

τ

$$\mu_l \quad = \quad \mu \quad + \quad \tau_l$$

(l mean populasi) (semua mean) (l pengaruh populasi)

- H_0: $\tau_l = \tau_2 = \ldots = \tau_g$

Respon X_{lj} berdistribusi $N(\mu + \tau_l, \sigma^2)$, dapat diekspresikan dalam bentuk

$$X_{lj} = \mu + \tau_l + e_{lj}$$

(semua mean) (pengaruh perlakuan) (error random)

$$x_{lj} = \bar{x} + (\bar{x}_l - \bar{x}) + (x_{lj} - \bar{x}_l)$$

(observasi) (overall sample mean) (estimasi) (residual)

TABEL ANOVA UNTUK PERBANDINGAN RATA-RATA POPULASI UNIVARIAT

Source of variation	Sum of squares (SS)	Degrees of freedom (d.f.)
Treatment	$SS_{tr} = \sum_{l=1}^{g} n_l \left(\bar{x}_l - \bar{x}\right)^2$	$g - 1$
Residual (error)	$SS_{res} = \sum_{l=1}^{g} \sum_{j=i}^{n_l} \left(x_{lj} - \bar{x}_l\right)^2$	$\sum_{l=1}^{g} n_l - g$
Total (corrected for the mean)	$SS_{cor} = \sum_{l=1}^{g} \sum_{j=i}^{n_l} \left(x_{lj} - \bar{x}\right)^2$	$\sum_{l=1}^{g} n_l - 1$

1 Way Multivariate Analysis of Variance (One Way MANOVA)

Model Linier :

$$X_{ij} = \mu + \tau_l + e_{lj}, \; j = 1,2,\ldots,n_l, \; l = 1,2,\ldots g$$

$$e_{lj} \sim \mathrm{NID}(0, \Sigma)$$

μ adalah rata - rata secara keseluruhan

τ_1 merupakan pengaruh perlakuan ke - 1

dengan $\sum_{l=1}^{g} n_1 \tau_l = 0$

- Univariat

$$x_{lj} \quad = \quad \bar{x} \quad + \quad (\bar{x}_l - \bar{x}) \quad + \quad (x_{lj} - \bar{x}_l)$$

(observasi) (overall sample mean) (estimasi) (residual)

- Multivariat

$$\sum_{l=1}^{g}\sum_{j=1}^{n_l}(x_{lj}-\bar{x})(x_{lj}-\bar{x})' = \sum_{l=1}^{g} n_l(\bar{x}_l-\bar{x})(\bar{x}_l-\bar{x})' + \sum_{l=1}^{g}\sum_{j=1}^{n_l}(x_{lj}-\bar{x}_l)(x_{lj}-\bar{x}_l)'$$

(total (corrected) sum) (treatment (between)) (residual (within) sum)

- Didefinisikan :

$$W = \sum_{l=1}^{g} \sum_{j=i}^{n_l} \left(x_{lj} - \overline{x}_l\right)\left(x_{lj} - \overline{x}_l\right)'$$

$$= \left(n_1 - 1\right) S_1 + \left(n_2 - 1\right) S_2 + \cdots + \left(n_g - 1\right) S_g$$

dimana S_l adalah sampel matriks kovarian untuk l sampel.

$$\sum_{l=1}^{g} \sum_{j=1}^{n_l} \left(x_{lj} - \overline{x}\right)\left(x_{lj} - \overline{x}\right)' = \sum_{l=1}^{g} n_l \left(\overline{x}_l - \overline{x}\right)\left(\overline{x}_l - \overline{x}\right)' + \sum_{l=1}^{g} \sum_{j=1}^{n_l} \left(x_{lj} - \overline{x}_l\right)\left(x_{lj} - \overline{x}_l\right)'$$

(total (corrected) sum) (treatment (between)) (residual (within) sum)

TABEL MANOVA UNTUK MEMBANDINGKAN POPULASI VEKTOR MEAN

Source of variation	Matrix of sum of squares and cross products (SSP)	Degrees of freedom (d.f.)
Treatment	$B = \sum_{l=1}^{g} n_l (\bar{x}_l - \bar{x})(\bar{x}_l - \bar{x})'$	$g - 1$
Residual (error)	$W = \sum_{l=1}^{g} \sum_{j=i}^{n_l} (x_{lj} - \bar{x}_l)(x_{lj} - \bar{x}_l)'$	$\sum_{l=1}^{g} n_l - g$
Total (corrected for the mean)	$B + W = \sum_{l=1}^{g} \sum_{j=i}^{n_l} (x_{lj} - \bar{x})(x_{lj} - \bar{x})'$	$\sum_{l=1}^{g} n_l - 1$

Uji pertama $H_0 : \tau_1 = \tau_2 = \cdots = \tau_g = 0$ menyatakan perumuman varians. Kita menolak H_0 jika rasio varians secara umum

$$\Lambda^* = \frac{|W|}{|B+W|} = \frac{\left| \sum_{l=1}^{g} \sum_{j=i}^{n_l} \left(x_{lj} - \bar{x}_l\right)\left(x_{lj} - \bar{x}_l\right)' \right|}{\left| \sum_{l=1}^{g} \sum_{j=i}^{n_l} \left(x_{lj} - \bar{x}\right)\left(x_{lj} - \bar{x}\right)' \right|}$$

kecil

Untuk kasus lain dengan sampel besar, modifikasi Λ^* dengan Bartlett dapat digunakan untuk uji H0

Bartlett menunjukkan bahwa jika H_0 benar dan $\sum n_l = n$ besar,

$$-\left(n-1-\frac{(p+g)}{2}\right)\ln \Lambda^* = -\left(n-1-\frac{(p+g)}{2}\right)\ln\left(\frac{|W|}{|B+W|}\right)$$

memiliki perkiraan distribusi chi-kuadrat dengan derajat kebebasan $p(g - 1)$. Konsekwensinya, untuk $\sum n_l = n$ besar, kita menolak H_0 pada taraf signifikansi α jika

$$-\left(n-1-\frac{(p+g)}{2}\right)\ln\left(\frac{|W|}{|B+W|}\right) > \chi^2_{p(g-1)}(\alpha)$$

Table 6.3 Distribution of Wilks' Lambda, $\Lambda^* = |\mathbf{W}|/|\mathbf{B}+\mathbf{W}|$

No. of variables	No. of groups	Sampling distribution for multivariate normal data
$p = 1$	$g \geq 2$	$\left(\frac{\Sigma n_\ell - g}{g-1}\right)\left(\frac{1-\Lambda^*}{\Lambda^*}\right) \sim F_{g-1, \Sigma n_\ell - g}$
$p = 2$	$g \geq 2$	$\left(\frac{\Sigma n_\ell - g - 1}{g-1}\right)\left(\frac{1-\sqrt{\Lambda^*}}{\sqrt{\Lambda^*}}\right) \sim F_{2(g-1), 2(\Sigma n_\ell - g - 1)}$
$p \geq 1$	$g = 2$	$\left(\frac{\Sigma n_\ell - p - 1}{p}\right)\left(\frac{1-\Lambda^*}{\Lambda^*}\right) \sim F_{p, \Sigma n_\ell - p - 1}$
$p \geq 1$	$g = 3$	$\left(\frac{\Sigma n_\ell - p - 2}{p}\right)\left(\frac{1-\sqrt{\Lambda^*}}{\sqrt{\Lambda^*}}\right) \sim F_{2p, 2(\Sigma n_\ell - p - 2)}$

Contoh 1 _univariat

Diambil sampel independen

- Populasi 1: 9, 6, 9
- Populasi 2: 0, 2
- Populasi 3 : 3, 1, 2

i. Susun hipotesis

$$H_0: \tau_1 = \tau_2 = \ldots = \tau_g$$

ii. Pilih tingkat signifikansi α=0,01

iii. Hitungan

$$x_{lj} \quad = \quad \bar{x} \quad + \quad (\bar{x}_l - \bar{x}) \quad + \quad (x_{lj} - \bar{x}_l)$$

$$\begin{pmatrix} 9 & 6 & 9 \\ 0 & 2 & \\ 3 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 4 & 4 \\ 4 & 4 & \\ 4 & 4 & 4 \end{pmatrix} + \begin{pmatrix} 4 & 4 & 4 \\ -3 & -3 & \\ -2 & -2 & -2 \end{pmatrix} + \begin{pmatrix} 1 & -2 & 1 \\ -1 & 1 & \\ 1 & -1 & 0 \end{pmatrix}$$

dimana

$$\bar{x} = (9+6+9+0+2+3+1+2)/8 = 4$$

$$\bar{x}_1 = (9+6+9)/3 = 8 \quad \bar{x}_2 = (0+2)/2 = 1 \qquad \bar{x}_1 = (3+1+2)/3 = 2$$

- Dari data y= [9,6,9,0,2,3,1,2]

$$x_{lj} \quad = \quad \bar{x} \quad + \quad (\bar{x}_l - \bar{x}) \quad + \quad (x_{lj} - \bar{x}_l)$$

$$\begin{pmatrix} 9 & 6 & 9 \\ 0 & 2 & \\ 3 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 4 & 4 \\ 4 & 4 & \\ 4 & 4 & 4 \end{pmatrix} + \begin{pmatrix} 4 & 4 & 4 \\ -3 & -3 & \\ -2 & -2 & -2 \end{pmatrix} + \begin{pmatrix} 1 & -2 & 1 \\ -1 & 1 & \\ 1 & -1 & 0 \end{pmatrix}$$

$$SS_{obs} = 9^2 + 6^2 + 9^2 + 0^2 + 2^2 + 3^2 + 1^2 + 2^2 = 216$$

$$SS_{mean} = 4^2 + 4^2 + 4^2 + 4^2 + 4^2 + 4^2 + 4^2 + 4^2 = 128$$

$$SS_{tr} = 4^2 + 4^2 + 4^2 + (-3)^2 + (-3)^2 + (-2)^2 + (-2)^2 + (-2)^2 = 78$$

$$SS_{res} = 1^2 + (-2)^2 + 1^2 + (-1)^2 + 1^2 + 1^2 + (-1)^2 + 0^2 = 10$$

$$SS_{obs} = SS_{mean} + SS_{tr} + SS_{res} \quad \text{atau} \quad 216 = 128 + 78 + 10$$

Tabel anava

Source of variation	Sum of squares (SS)	Degrees of freedom (d.f.)
Treatment	$SS_{tr} = 78$	$g - 1 = 3 - 1 = 2$
Residual (error)	$SS_{res} = 10$	$\sum_{l=1}^{g} n_l - g = (3+2+3) - 3 = 5$
Total (corrected for the mean)	$SS_{cor} = 88$	$\sum_{l=1}^{g} n_l - 1 = 7$

iv. Keputusan Uji

$$F = \frac{SS_{tr}/(g-1)}{SS_{res}/\left(\sum_{l=1}^{g} n_l - g\right)} = \frac{78/2}{10/5} = 19.5$$

$F = 19.5 > F_{2,5}(.01) = 13.27$. Tolak $H_0 : \tau_1 = \tau_2 = \cdots = \tau_g = 0$

Contoh 2_1 way manova
Diketahui sampel dengan ukuran
$n_1 = 3$, $n_2 = 2$, dan $n_3 = 3$.

$$\begin{pmatrix} \begin{bmatrix} 9 \\ 3 \end{bmatrix} & \begin{bmatrix} 6 \\ 2 \end{bmatrix} & \begin{bmatrix} 9 \\ 7 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 4 \end{bmatrix} & \begin{bmatrix} 2 \\ 0 \end{bmatrix} & \\ \begin{bmatrix} 3 \\ 8 \end{bmatrix} & \begin{bmatrix} 1 \\ 9 \end{bmatrix} & \begin{bmatrix} 2 \\ 7 \end{bmatrix} \end{pmatrix}$$

$$\bar{x}_1 = \begin{bmatrix} 8 \\ 4 \end{bmatrix}, \bar{x}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \bar{x}_3 = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$$

$$\bar{x} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

p=2

g=3

i. $H_0 : \tau_1 = \tau_2 = \tau_3$

$H_1 :$ setidaknya ada satu pasang dimana $\tau_j \neq \tau_l$

ii. Misal tingkat signifikansi 10%

iii. Hitungan

Akan dicari SS mean , SS treat , dan SS res pada variabel pertama dengan univariat ANOVA

$$\begin{pmatrix} 9 & 6 & 9 \\ 0 & 2 & \\ 3 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 4 & 4 \\ 4 & 4 & \\ 4 & 4 & 4 \end{pmatrix} + \begin{pmatrix} 4 & 4 & 4 \\ -3 & -3 & \\ -2 & -2 & -2 \end{pmatrix} + \begin{pmatrix} 1 & -2 & 1 \\ -1 & 1 & \\ 1 & -1 & 0 \end{pmatrix}$$

$$SS_{obs} = SS_{mean} + SS_{tr} + SS_{res}$$

$$216 = 128 + 78 + 10$$

$$\text{Total SS(corrected)} = SS_{obs} - SS_{mean} = 216 - 128 = 88$$

Re...

$$SS_{obs} = 9^2 + 6^2 + 9^2 + 0^2 + 2^2 + 3^2 + 1^2 + 2^2 = 216$$

$$SS_{mean} = 4^2 + 4^2 + 4^2 + 4^2 + 4^2 + 4^2 + 4^2 + 4^2 = 128$$

$$SS_{tr} = 4^2 + 4^2 + 4^2 + (-3)^2 + (-3)^2 + (-2)^2 + (-2)^2 + (-2)^2 = 78$$

$$SS_{res} = 1^2 + (-2)^2 + 1^2 + (-1)^2 + 1^2 + 1^2 + (-1)^2 + 0^2 = 10$$

$$SS_{obs} = SS_{mean} + SS_{tr} + SS_{res} \quad \text{atau} \quad 216 = 128 + 78 + 10$$

Akan dicari SS mean , SS treat , dan SS res pada variabel kedua dengan univariat ANOVA

$$\begin{pmatrix} 3 & 2 & 7 \\ 4 & 0 & \\ 8 & 9 & 7 \end{pmatrix} = \begin{pmatrix} 5 & 5 & 5 \\ 5 & 5 & \\ 5 & 5 & 5 \end{pmatrix} + \begin{pmatrix} -1 & -1 & -1 \\ -3 & -3 & \\ 3 & 3 & 3 \end{pmatrix} + \begin{pmatrix} -1 & -2 & 3 \\ 2 & -2 & \\ 0 & 1 & -1 \end{pmatrix}$$

$$SS_{obs} = SS_{mean} + SS_{tr} + SS_{res}$$

$$272 = 200 + 48 + 24$$

$$\text{Total SS(corrected)} = SS_{obs} - SS_{mean} = 272 - 200 = 72$$

$$\begin{pmatrix} 9 & 6 & 9 \\ 0 & 2 & \\ 3 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 4 & 4 \\ 4 & 4 & \\ 4 & 4 & 4 \end{pmatrix} + \begin{pmatrix} 4 & 4 & 4 \\ -3 & -3 & \\ -2 & -2 & -2 \end{pmatrix} + \begin{pmatrix} 1 & -2 & 1 \\ -1 & 1 & \\ 1 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 2 & 7 \\ 4 & 0 & \\ 8 & 9 & 7 \end{pmatrix} = \begin{pmatrix} 5 & 5 & 5 \\ 5 & 5 & \\ 5 & 5 & 5 \end{pmatrix} + \begin{pmatrix} -1 & -1 & -1 \\ -3 & -3 & \\ 3 & 3 & 3 \end{pmatrix} + \begin{pmatrix} -1 & -2 & 3 \\ 2 & -2 & \\ 0 & 1 & -1 \end{pmatrix}$$

Mean: 4(5) + 4(5) + ... + 4(5) = 160

Treatment: 3(4)(-1) + 2(-3)(-3) + 3(-2)(3) = -12

Residual: 1(-1) + (-2) (-2) + (1)(3)+ (-1)(2) + ... + 0(-1) = 1

Total: 9(3) + 6(2) + 9(7) + 0(4) + ... + 2(7) = 149

Total (corrected) perkalian = total perkalian – perkalian rata-rata

= 149 – 160 = -11

Tabel MANOVA

Source of variation	Matrix of sum of squares and cross products (SSP)	Degrees of freedom (d.f.)
Treatment	$\begin{bmatrix} 78 & -12 \\ -12 & 48 \end{bmatrix}$	$3-1=2$
Residual (error)	$\begin{bmatrix} 10 & 1 \\ 1 & 24 \end{bmatrix}$	$3+2+3-3=5$
Total (corrected for the mean)	$\begin{bmatrix} 88 & -11 \\ -11 & 72 \end{bmatrix}$	7

Mean: 4(5) + 4(5) + ... + 4(5) = 160

Treatment: 3(4)(-1) + 2(-3)(-3) + 3(-2)(3) = -12

Residual: 1(-1) + (-2) (-2) + (1)(3)+ (-1)(2) + ... + 0(-1) = 1

Total: 9(3) + 6(2) + 9(7) + 0(4) + ... + 2(7) = 149

Total (corrected) perkalian = total perkalian – perkalian rata-rata

= 149 – 160 = -11

$$SS_{obs} = SS_{mean} + SS_{tr} + SS_{res}$$
$$216 = 128 + 78 + 10$$
$$\text{Total SS(corrected)} = SS_{obs} - SS_{mean} = 216 - 128 = 88$$

$$SS_{obs} = SS_{mean} + SS_{tr} + SS_{res}$$
$$272 = 200 + 48 + 24$$
$$\text{Total SS(corrected)} = SS_{obs} - SS_{mean} = 272 - 200 = 72$$

Perhatikan bahwa

$$\begin{bmatrix} 88 & -11 \\ -11 & 72 \end{bmatrix} = \begin{bmatrix} 78 & -12 \\ -12 & 48 \end{bmatrix} + \begin{bmatrix} 10 & 1 \\ 1 & 24 \end{bmatrix}$$

$$\Lambda^* = \frac{|W|}{|B+W|} = \frac{\begin{vmatrix} 10 & 1 \\ 1 & 24 \end{vmatrix}}{\begin{vmatrix} 88 & -11 \\ -11 & 72 \end{vmatrix}} = \frac{10(24)-(1)^2}{88(72)-(-11)^2} = \frac{239}{6215} = 0.0385$$

$p = 2$ dan $g = 3$, pada tabel distribusi WILKS' LAMBDA mengindikasikan bahwa uji memerlukan $H_0 : \tau_1 = \tau_2 = \cdots = \tau_g = 0$ dan H_1: paling sedikit satu tanda = tidak berlaku.

$$\left(\frac{\sum n_l - g - 1}{g - 1}\right)\left(\frac{1-\sqrt{\Lambda^*}}{\sqrt{\Lambda^*}}\right) = \left(\frac{1-\sqrt{.0385}}{\sqrt{.0385}}\right)\left(\frac{8-3-1}{3-1}\right) = 8.19$$

$$8.19 > F_{2(g-1),2(\sum n_l - g - 1)} = F_{4,8}(0.1) = 7.01$$

Artinya Ho ditolak pada $\alpha = 0.1$ dan kesimpulannya bahwa ada perbedaan perlakuan.

Contoh 3

Departemen Kesehatan dan Sosial melakukan riset untuk mengetahui pengaruh status RS (swasta atau negeri) terhadap biaya. 4 komponen biaya ditetapkan sebagai berikut X1: biaya perawat, X2: biaya lab, X3: biaya operasi, X4: biaya rumah tangga rs. Sebanyak 516 pengamatan dilakukan untuk setiap p = 4 variabel biaya terhadap g=3 kel.

Kelompok	Jumlah Pengamatan	Vektor Rata-Rata Sampel
i = 1 (Pribadi)	$n_1 = 271$	$\bar{x}_1 = \begin{bmatrix} 2.066 \\ 0.480 \\ 0.082 \\ 0.360 \end{bmatrix}$
i = 2 (Sukarela)	$n_2 = 138$	$\bar{x}_2 = \begin{bmatrix} 2.167 \\ 0.596 \\ 0.124 \\ 0.418 \end{bmatrix}$
i = 3 (Pemerintah)	$n_3 = 107$	$\bar{x}_3 = \begin{bmatrix} 2.273 \\ 0.521 \\ 0.125 \\ 0.383 \end{bmatrix}$
$\sum_{i=1}^{3} n_i = 516$		

i. Susun Hipotesis

$$H_0 : \tau_1 = \tau_2 = \tau_3$$ (tidak ada pengaruh kepemilikan RS thd biaya)

$$H_1 : \tau_i = \tau_j, \text{untuk suatu}\ i \neq j$$

ii. Pilih tingkat signifikansi 0,01

iii. Hitungan

- Misal telah dihitung matrik kovarians-nya untuk sampel nya adalah

$$S_1 = \begin{bmatrix} 0.291 & & & \\ -0.001 & 0.011 & & \\ 0.002 & 0.000 & 0.001 & \\ 0.019 & 0.003 & 0.000 & 0.010 \end{bmatrix} \quad S_2 = \begin{bmatrix} 0.561 & & & \\ 0.011 & 0.025 & & \\ 0.001 & 0.004 & 0.005 & \\ 0.037 & 0.007 & 0.002 & 0.019 \end{bmatrix}$$

$$S_3 = \begin{bmatrix} 0.261 & & & \\ 0.030 & 0.017 & & \\ 0.003 & -0.000 & 0.004 & \\ 0.018 & 0.006 & 0.001 & 0.013 \end{bmatrix}$$

- diperoleh

$$\mathbf{W} = (n_1 - 1)S_1 + (n_2 - 1)S_2 + (n_3 - 1)S_3$$

$$= \begin{bmatrix} 182.962 & & & \\ 4.408 & 8.200 & & \\ 1.695 & 0.633 & 1.484 & \\ 9.581 & 2.428 & 0.394 & 6.538 \end{bmatrix}$$

$$\bar{\mathbf{x}} = \frac{n_1\bar{\mathbf{x}}_1 + n_2\bar{\mathbf{x}}_2 + n_3\bar{\mathbf{x}}_3}{n_1 + n_2 + n_3} = \begin{bmatrix} 2.136 \\ 0.519 \\ 0.102 \\ 0.380 \end{bmatrix}$$

$$\mathbf{B} = \sum_{\ell=1}^{3} n_\ell(\bar{\mathbf{x}}_\ell - \bar{\mathbf{x}})(\bar{\mathbf{x}}_\ell - \bar{\mathbf{x}})' = \begin{bmatrix} 3.475 & & & \\ 1.111 & 1.225 & & \\ .821 & .453 & .235 & \\ .584 & .610 & .230 & .304 \end{bmatrix}$$

Table 6.3 Distribution of Wilks' Lambda, $\Lambda^* = |\mathbf{W}|/|\mathbf{B}+\mathbf{W}|$

No. of variables	No. of groups	Sampling distribution for multivariate normal data
$p = 1$	$g \geq 2$	$\left(\frac{\Sigma n_\ell - g}{g-1}\right)\left(\frac{1-\Lambda^*}{\Lambda^*}\right) \sim F_{g-1, \Sigma n_\ell - g}$
$p = 2$	$g \geq 2$	$\left(\frac{\Sigma n_\ell - g - 1}{g-1}\right)\left(\frac{1-\sqrt{\Lambda^*}}{\sqrt{\Lambda^*}}\right) \sim F_{2(g-1), 2(\Sigma n_\ell - g - 1)}$
$p \geq 1$	$g = 2$	$\left(\frac{\Sigma n_\ell - p - 1}{p}\right)\left(\frac{1-\Lambda^*}{\Lambda^*}\right) \sim F_{p, \Sigma n_\ell - p - 1}$
$p \geq 1$	$g = 3$	$\left(\frac{\Sigma n_\ell - p - 2}{p}\right)\left(\frac{1-\sqrt{\Lambda^*}}{\sqrt{\Lambda^*}}\right) \sim F_{2p, 2(\Sigma n_\ell - p - 2)}$

p=4 pada g=3

Dapat dihitung

$$\Lambda^* = \frac{|\mathbf{W}|}{|\mathbf{B}+\mathbf{W}|} = .7714$$

$$\left(\frac{\Sigma n_\ell - p - 2}{p}\right)\left(\frac{1-\sqrt{\Lambda^*}}{\sqrt{\Lambda^*}}\right) = \left(\frac{516 - 4 - 2}{4}\right)\left(\frac{1-\sqrt{.7714}}{\sqrt{.7714}}\right) = 17.67$$

Keputusan Uji

$$F_{2(4),2(510)}(0{,}01) = \chi_8^2(0{,}01)/8 = 2{,}51$$

$$17{,}67 > F_{8,1020}(0{,}01) = 2{,}51$$

Maka H0 ditolak → rata-rata biaya RS tergantung dari status kepemilikan RS

Karena cth di atas jumlah n besar maka bisa menggunakan alternatif :

$$-(n-1-(p+g)/2)\ln\left(\frac{|\mathbf{W}|}{|\mathbf{B}+\mathbf{W}|}\right) = -511.5\ln(.7714) = 132.76$$

$$\chi^2_{p(g-1)}(0{,}01) = \chi_8^2(0{,}01) = 20{,}09$$

$$\text{karena } 132{,}76 > \chi_8^2(0{,}01) \text{ maka H0 ditolak}$$