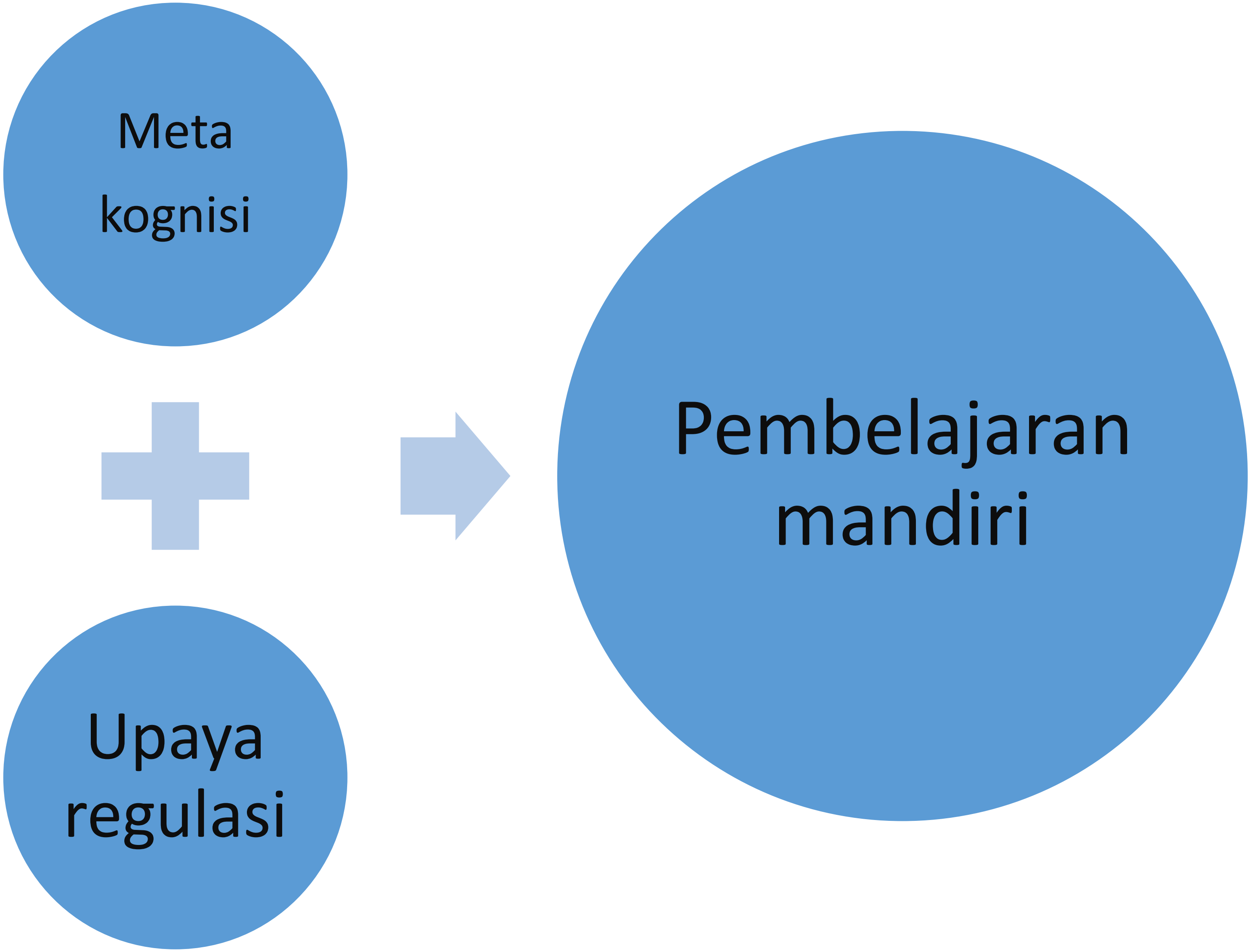
Analisis Regresi Berganda

Ilustrasi penggunan analisis regresi berganda…

- Penelitian pendidikan ”*Self regulated Learning Strategies and Achievement in an Introduction to Information Systems Course*” (Catherine S Chen, 2002, *Information Technology, Learning and Performance journal* Vol 20 No 1, *College of Bussiness Ball State University,*) meneliti tentang pembelajaran mandiri yang mungkin dipengaruhi oleh regulasi metakognisi, waktu dan lingkungan pembelajaran, upaya regulasi, pembelajaran *peer*, pengalaman IT, penggunaan software.

us di atas kira-kira pake

atistika apa ya?


Meta kognisi
Upaya regulasi
Pembelajaran mandiri

Model Linier anreg berganda...

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots \beta_k x_{ik} + \varepsilon_i$$

$$= \beta_0 + \sum_{j=1}^{k} \beta_j x_{ij} + \varepsilon_i, \qquad i = 1,2,\ldots,n$$

Dgn Metode Kuadrat Terkecil (MKT), bentuk L untuk mengestimasi parameter

$$\mathrm{L} = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{j=1}^{k} \beta_j x_{ij} \right)^2 \Rightarrow\Rightarrow \frac{\partial \mathrm{L}}{\partial \beta_0}, \ldots, \frac{\partial \mathrm{L}}{\partial \beta_j}$$

Sehingga dapat diperoleh

$$n\hat{\beta}_0 + \hat{\beta}_1 \sum_{i=1}^{n} x_{i1} + \hat{\beta}_2 \sum_{i=1}^{n} x_{i2} + \cdots + \hat{\beta}_k \sum_{i=1}^{n} x_{ik} = \sum_{i=1}^{n} y_i$$

$$\hat{\beta}_0 \sum_{i=1}^{n} x_{i1} + \hat{\beta}_1 \sum_{i=1}^{n} x_{i1}^2 + \hat{\beta}_2 \sum_{i=1}^{n} x_{i1} x_{i2} + \cdots + \hat{\beta}_k \sum_{i=1}^{n} x_{i1} x_{ik} = \sum_{i=1}^{n} x_{i1} y_i$$

$$\vdots$$

$$\hat{\beta}_0 \sum_{i=1}^{n} x_{ik} + \hat{\beta}_1 \sum_{i=1}^{n} x_{ik} x_{i1} + \hat{\beta}_2 \sum_{i=1}^{n} x_{ik} x_{i2} + \cdots + \hat{\beta}_k \sum_{i=1}^{n} x_{ik}^2 = \sum_{i=1}^{n} x_{ik} y_i$$

Alternatif .. dalam bentuk matriks ...

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots \beta_k x_{ik} + \varepsilon_i$$

$$= \beta_0 + \sum_{j=1}^{k} \beta_j x_{ij} + \varepsilon_i, \qquad i = 1,2,\ldots,n$$

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

Dengan

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} 1 & x_{11} & \ldots & x_{1k} \\ 1 & x_{21} & \ldots & x_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} & x_{nk} \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix}, \quad \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

$$L = \sum_{i=1}^{n} \boldsymbol{\varepsilon}_i^2 = \boldsymbol{\varepsilon}'\boldsymbol{\varepsilon} = (\mathbf{y} - \mathbf{X}\boldsymbol{\beta})'(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}) \Rightarrow \frac{\partial L}{\partial \boldsymbol{\beta}} = 0$$

$$\Rightarrow \hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$$

So…

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_{i1} & \sum_{i=1}^{n} x_{i2} & \cdots & \sum_{i=1}^{n} x_{ik} \\ \sum_{i=1}^{n} x_{i1} & \sum_{i=1}^{n} x_{i1}^2 & \sum_{i=1}^{n} x_{i1}x_{i2} & \cdots & \sum_{i=1}^{n} x_{i1}x_{ik} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \sum_{i=1}^{n} x_{ik} & \sum_{i=1}^{n} x_{ik}x_{i1} & \sum_{i=1}^{n} x_{ik}x_{i2} & \cdots & \sum_{i=1}^{n} x_{ik}^2 \end{bmatrix} \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \vdots \\ \hat{\beta}_k \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_{i1}y_i \\ \vdots \\ \sum_{i=1}^{n} x_{ik}y_i \end{bmatrix}$$

Contoh sblmnya…

No	Meta kognisi	Upaya Reg	Pemb. Mandiri	No	Meta kognisi	Upaya Reg	Pemb. Mandiri
1	13	11	9	21	12	12	12
2	14	13	11	22	14	12	12
3	14	13	13	23	15	14	15
4	14	13	11	24	12	11	10
5	13	10	8	25	13	12	10
6	14	12	11	26	12	14	13
7	14	13	11	27	19	14	14
8	15	10	11	28	15	11	12
9	14	12	11	29	14	10	11
10	11	7	9	30	16	11	11
11	16	13	11	31	14	12	13
12	15	13	11	32	12	8	10
13	15	12	12	33	13	10	11
14	16	14	12	34	16	12	12
15	11	9	6	35	16	10	14
16	10	14	11	36	16	12	11
17	16	15	15	37	12	13	11
18	14	13	12	38	11	9	7
19	16	12	10	39	12	13	12
20	12	12	11	40	16	14	12

$$\mathbf{X} = \begin{bmatrix} 1 & 13 & 11 \\ 1 & 14 & 13 \\ 1 & 14 & 13 \\ 1 & 14 & 13 \\ 1 & 13 & 10 \\ 1 & 14 & 12 \\ 1 & 14 & 13 \\ 1 & 15 & 10 \\ \vdots & \vdots & \vdots \\ 1 & 16 & 14 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 9 \\ 11 \\ 13 \\ 11 \\ 8 \\ 11 \\ 11 \\ 11 \\ \vdots \\ 12 \end{bmatrix}$$

$$\mathbf{X'X} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 13 & 14 & \cdots & 16 \\ 11 & 13 & \cdots & 14 \end{bmatrix},$$

$$\mathbf{X'y} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 13 & 14 & \cdots & 16 \\ 11 & 13 & \cdots & 14 \end{bmatrix} \begin{bmatrix} 12 \\ 12 \\ \vdots \\ 12 \end{bmatrix}$$

$$\text{tentukan } \hat{\boldsymbol{\beta}} = (\mathbf{X'X})^{-1}\mathbf{X'y}$$

Uji signifikansi regresi ganda

→ Uji untuk menentukan apakah ada hubungan linier antara variabel respon y dengan regresor x1,x2,x3,…,xk

Langkah-langkah :

i. $$H_0 : \beta_1 = \beta_2 = \cdots = \beta_k = 0$$
$$H_1 : \beta_j \neq 0, \text{setidaknya satu } j$$

ii. Pilih α

iii. Susun tabel ANAVA

SV	JK	db	RK	F0
Regresi	JKR	k	RKR	RKR/RKS
Sesatan	JKS	n-p	RKS	
Total	JKT	n-1		

iv. Tolak H0 jika F0>Ftabel=Fα,k,n-p

Dengan
$$JK_S = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = \sum_{i=1}^{n} e_i^2 = \mathbf{e'e}$$

substitusi
$$\mathbf{e} = \mathbf{y} - \hat{\mathbf{y}} = \mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}$$

Akan diperoleh :

$$JK_S = \mathbf{y'y} - \hat{\boldsymbol{\beta}}'\mathbf{X'y}$$

$$JK_T = \sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n} = \mathbf{y'y} - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}$$

$$JK_S = \mathbf{y'y} - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n} - \left[\hat{\boldsymbol{\beta}}'\mathbf{X'y} - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}\right]$$

$$JK_R = \hat{\boldsymbol{\beta}}'\mathbf{X'y} - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}$$

Contoh sebelumnya

Langkah-langkah :

i.
$$H_0 : \beta_1 = \beta_2 = \cdots = \beta_k = 0$$
$$H_1 : \beta_j \neq 0, \text{setidaknya satu } j$$

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	.325	1.768		.184	.855
	metakognisi	.340	.119	.353	2.848	.007
	upaya_reg	.519	.127	.506	4.087	.000

a. Dependent Variable: Pemb_mandiri

ii. Pilih α=5%

iii. Susun tabel ANAVA

ANOVA[b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	39.612	1	39.612	16.475	.000[a]
	Residual	91.363	38	2.404		
	Total	130.975	39			

a. Predictors: (Constant), metakognisi

b. Dependent Variable: Pemb_mandiri

iv. Model $y(\text{pemb.mandiri}) = 0.325 + 0.340 x_1 (\text{metakognisi}) + 0.519 x_2 (\text{upaya\_reg})$

Dapat digunakan

R^2 dan R^2 adjusted

R-squared disebut juga dengan koefisien determinasi sebagai ukuran statistika kecocokan dengan model, dirumuskan :

$$R^2 = \frac{JK_R}{JK_T} = 1 - \frac{JK_S}{JK_T}$$

Contoh sebelumnya;

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.721 [a]	.519	.493	1.304

a. Predictors: (Constant), upaya_reg, metakognisi

R-squared=0.519 menunjukkan sebesar 51.9%
Model menerangkan variabilitas variabel respon sebesar 51.9%

Problem.

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.550[a]	.302	.284	1.551

a. Predictors: (Constant), metakognisi

Masalah → R-suared bertambah ketika regresor bertambah, sulit untuk menentukan kenaikan tsb karena penambahan regresor

→ Alternatif : menggunakan adjusted R-squared

$$R^2{}_{\text{adj}} = 1 - \frac{JK_S/(n-p)}{JK_T/(n-1)}$$