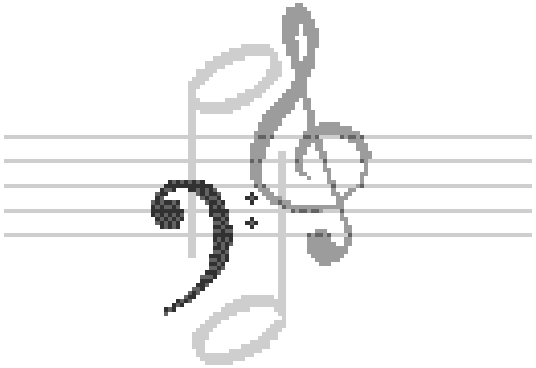


Inferensi dalam Analisis Regresi dan Korelasi

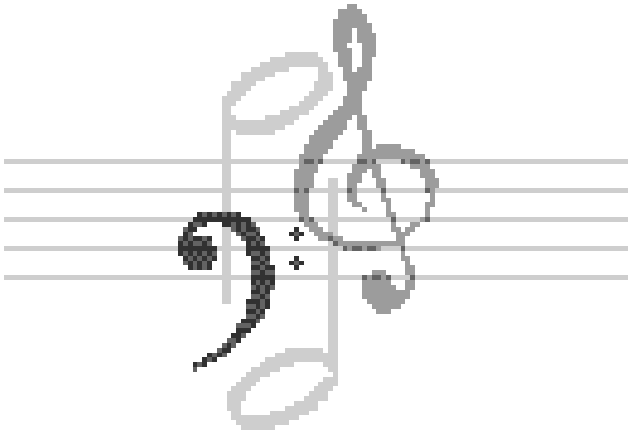
MENGUJI KOEFISIEN REGRESI DENGAN ANALISIS VARIANSI

Perhatikan

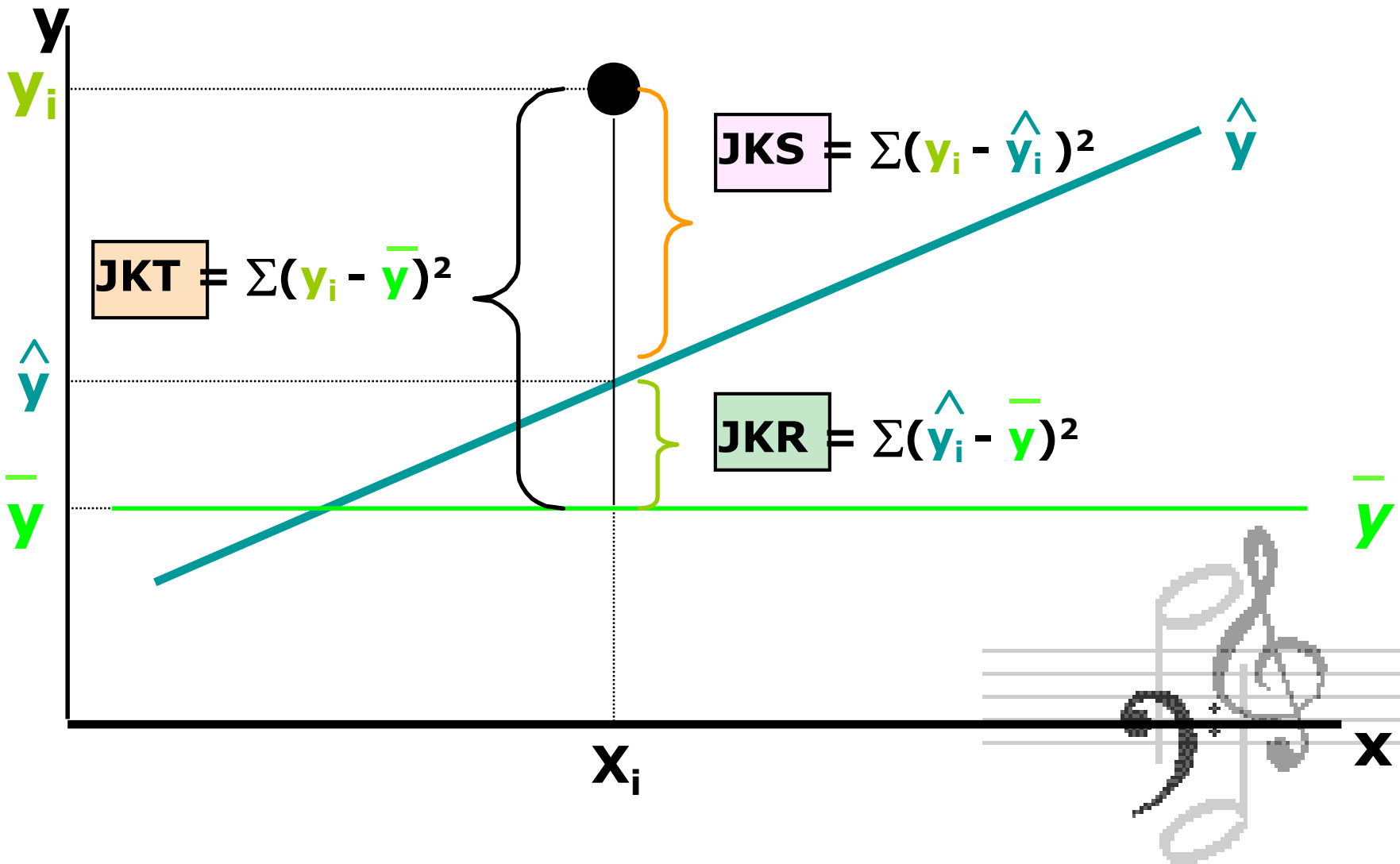
$$\underbrace{(y_i - \bar{y})}_{\text{Total}} = \underbrace{(\hat{y}_i - \bar{y})}_{\text{regresi}} + \underbrace{(y_i - \hat{y}_i)}_{\text{sisa}}$$

$$\underbrace{\sum_{i=1}^{n}(y_i - \bar{y})^2}_{JK_T} = \underbrace{\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2}_{JK_R} + \underbrace{2\sum_{i=1}^{n}(\hat{y}_i - \bar{y})(y_i - \hat{y}_i)}_{=0 \text{ (buktikan !!!)}} + \underbrace{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}_{JK_S}$$

Tentukan JKT dan JKR !



Variasi yang diterangkan dan Yang tidak dapat diterangkan



Contoh, utk model

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

i. $H_0 : \beta_1 = 0$

$H_1 : \beta_1 \neq 0$

ii. Tingkat signifikansi 5%

iii. Tabel ANAVA

Sumber Variasi	JK	dk	RK	F0
Regresi	JKR= $b^2 \sum_{i=1}^{n} (x_i - \bar{x})^2$	1	RKR =JKR/1	F=RKR/RKS
Sesatan	JKS= JKT-JKR	n-2	RKS =JKS/n-2	Ftabel F(α, 1,n-2)
Total	JKT= $\sum_{i=1}^{n} y_i^2 - \frac{(\sum y_i)^2}{n}$	n-1		

DK. Tolak H0 jika F0>Ftabel =Fα,1,n-2

Ada hubungan linier antara Mat dan Fis

Koefisien Determinasi, r^2

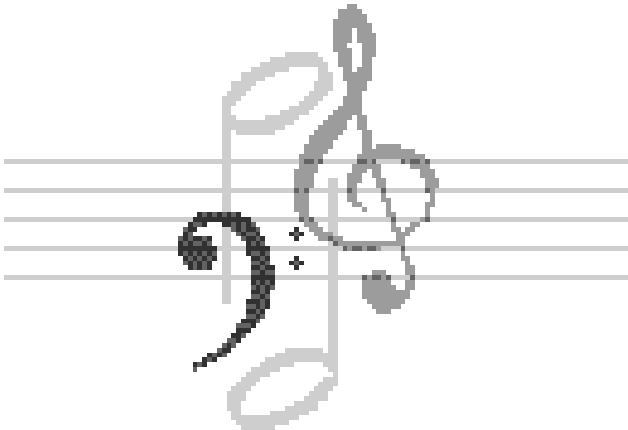
- Koefisien Determinasi adalah bagian dari variasi total dalam variabel dependen yang dijelaskan oleh variasi dalam variabel independen
- Disebut juga dengan r-squared dan dinotasikan dengan r^2

$$r^2 = \frac{JK_R}{JK_T} = \frac{\text{Jumlah kuadrat yang dijelaskan oleh regresi}}{\text{Jumlah kuadrat total}}$$

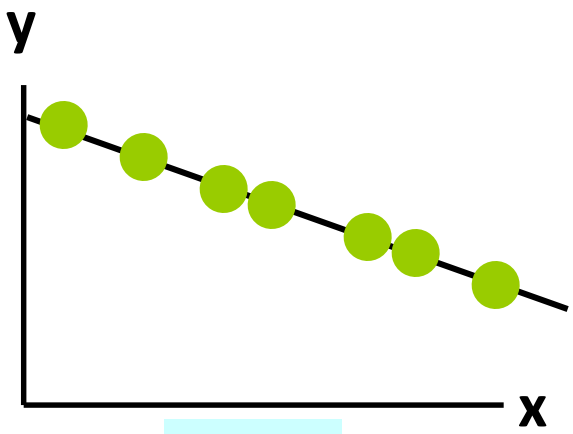
dengan $0 \le r^2 \le 1$

$$r = \pm\sqrt{r^2}, \quad -1 \le r \le 1$$

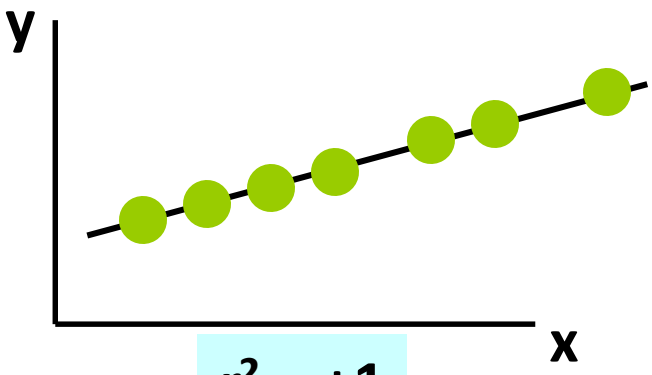
r = Koefisien Korelasi



Gambaran nilai r^2



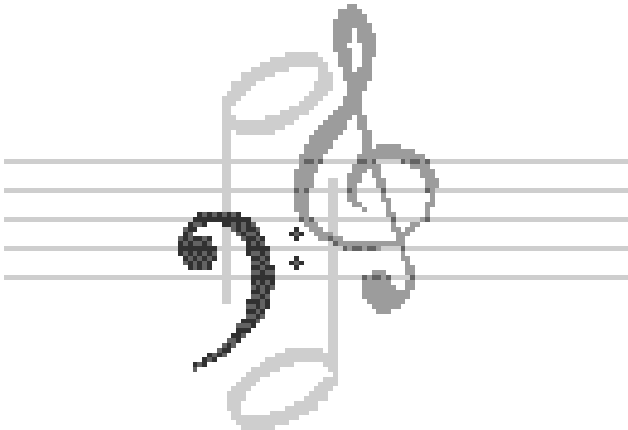
$r^2 = 1$



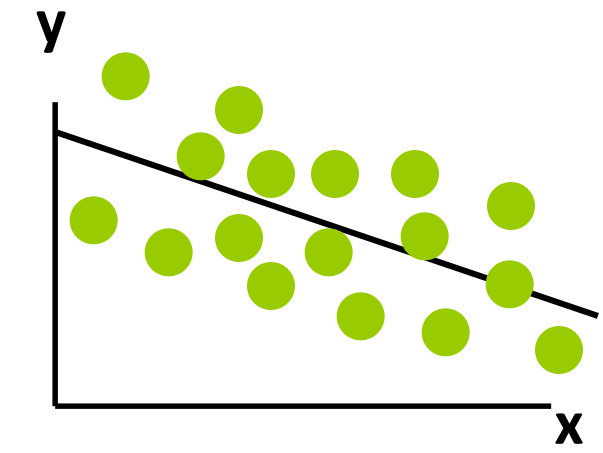
$r^2 = +1$

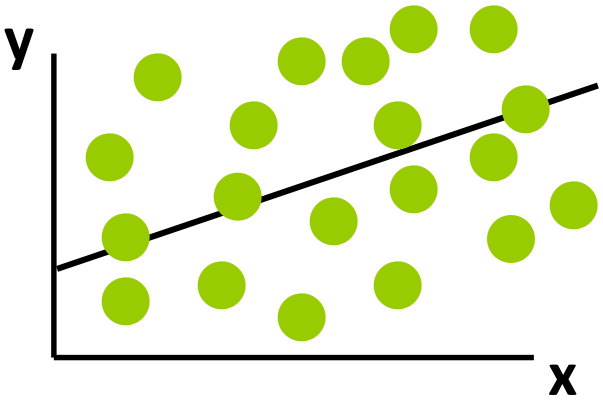
$r^2 = 1$

Hubungan linier sempurna antara x dan y :
100% variasi dalam y dijelaskan oleh variasi dalam x



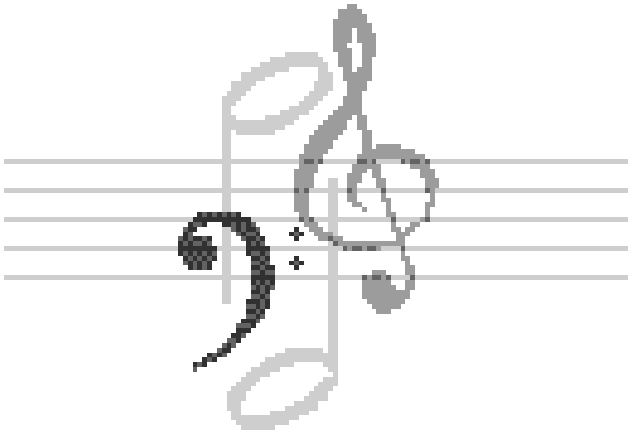
Gambaran nilai r^2



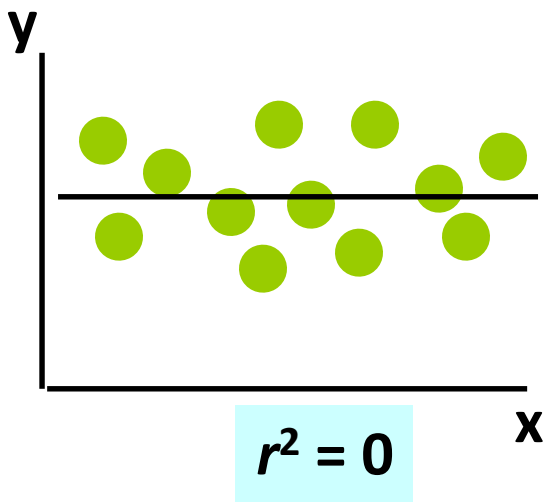


$0 < r^2 < 1$

Hubungan linier antara x dan y lemah :
Beberapa tapi tidak semua variasi y dijelaskan oleh variasi dalam x



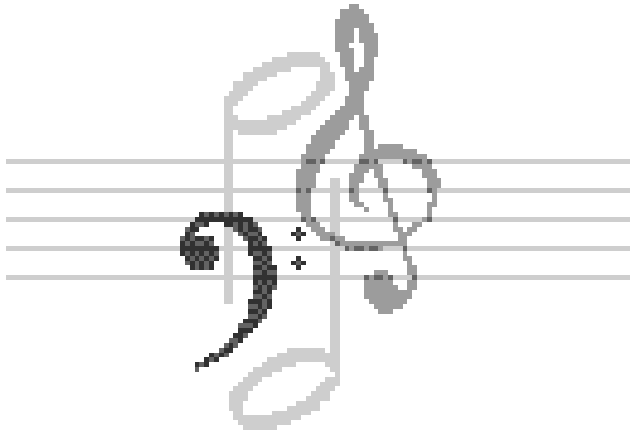
Gambaran nilai r^2



$r^2 = 0$

Tidak ada hubungan linier antara x dan y

Nilai Y tidak tergantung x



Contoh yang lalu

Mat (X)	Fis (Y)	XY	X2	Y2
60	80	4800	3600	6400
45	69	3105	2025	4761
50	71	3550	2500	5041
60	85	5100	3600	7225
50	80	4000	2500	6400
65	82	5330	4225	6724
60	89	5340	3600	7921
65	93	6045	4225	8649
50	76	3800	2500	5776
65	86	5590	4225	7396
45	71	3195	2025	5041
50	69	3450	2500	4761
665	951	53305	37525	76095

$a = 29.529, \quad b = 0.897$

$$\mathrm{JK_R} = b^2 \sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2 = 541.693$$

$$\mathrm{JK_T} = \sum_{i=1}^{n} y_i^2 - \frac{\left(\sum y_i\right)^2}{n} = 728.25$$

$$\mathrm{JK_S} = \mathrm{JK_T} - \mathrm{JK_R} = 728.25 - 541.693 = 186.557$$

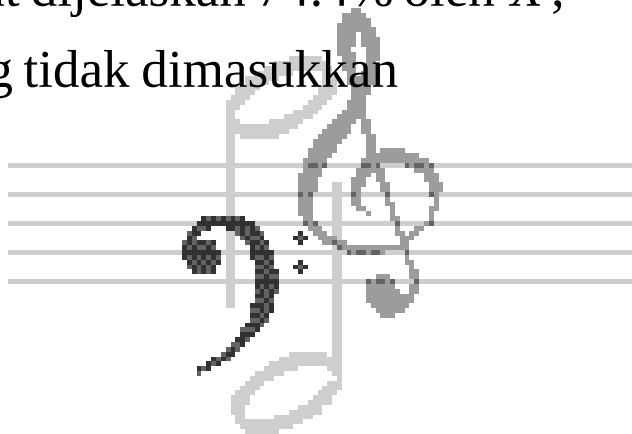
Jadi bisa kita hitung

$$r^2 = \frac{\mathrm{JK_R}}{\mathrm{JK_T}} = \frac{541.693}{728.25} = 0.743828$$

jadi persamaan regresi $\hat{Y}_i = 29.5294 + 0.8972 X_i$, nilai Y dapat dijelaskan 74.4% oleh X, sedangkan sisanya 25.6% diterangkan oleh variabel lain yang tidak dimasukkan dalam model

Total variasi dalam Y diturunkan sebanyak 74,4% oleh X

$$r = \sqrt{0.744} = 0.863$$



Kesalahan Baku Taksiran

(Standard Error of Estimate)

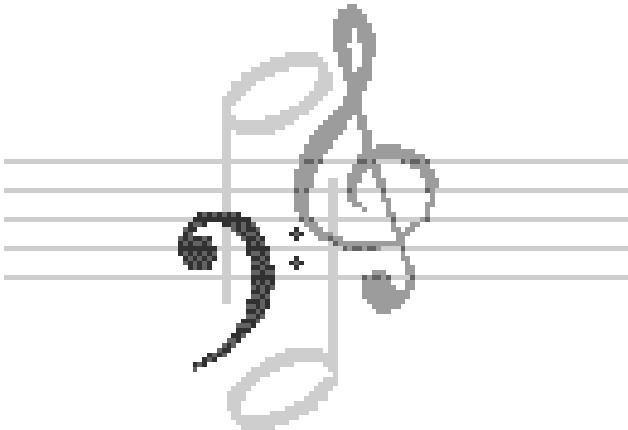
- Merupakan ukuran variabilitas antara Y dengan nilai Y prediksi

$$s_{y.x} = \sqrt{\frac{\mathrm{JK_S}}{\mathrm{n-2}}}$$

- Contoh yll:

$$\mathrm{JK_S} = 186.557$$

$$s_{y.x} = \sqrt{\frac{186.557}{12-2}} = 4.319$$



Kesalahan Baku Koef. Regresi

definisi

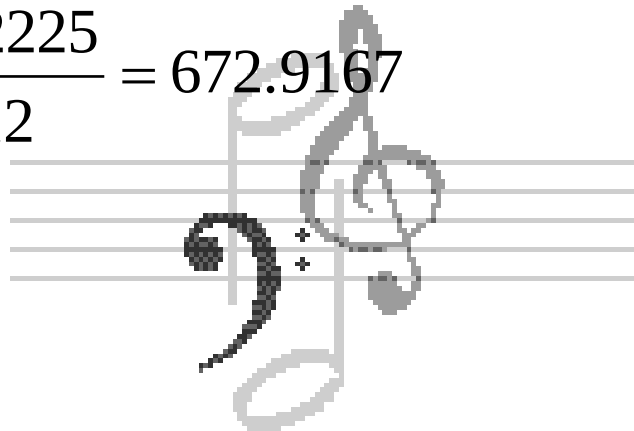
$$s_b = \sqrt{\frac{s_{y.x}^2}{c}}, \quad c = \sum x^2 - \frac{\left(\sum x\right)^2}{n}$$

Contoh yll

$$\sum x^2 = 37525, \quad \left(\sum x\right)^2 = (665)^2 = 442225$$

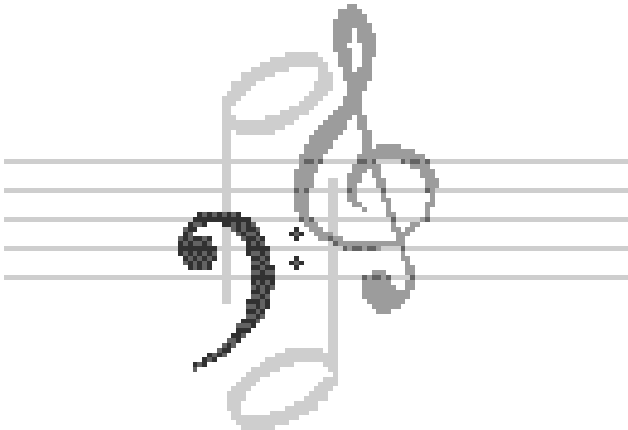
$$s_b = \sqrt{\frac{s_{y.x}^2}{c}}, \quad c = \sum x^2 - \frac{\left(\sum x\right)^2}{n} = 37525 - \frac{442225}{12} = 672.9167$$

$$s_b = \sqrt{\frac{s_{y.x}^2}{c}} = \sqrt{\frac{18.6557}{672.9167}} = 0.166504$$



Persyaratan pada uji regresi linier

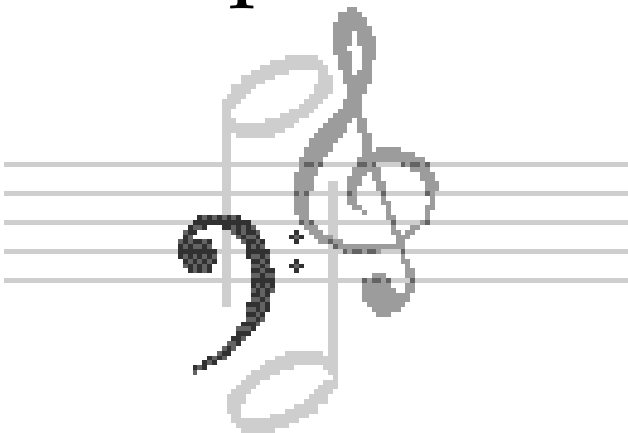
1. Normalitas
2. Linieritas dan Keberartian
3. Independensi
4. Homoskedastisitas



Uji linieritas

1. Susun hipotesis
2. Pilih tingkat signifikansi
3. Hitung anava
4. Kesimpulan :

   Tolak Ho jika Fobs>Fk-2,n-k,alpha

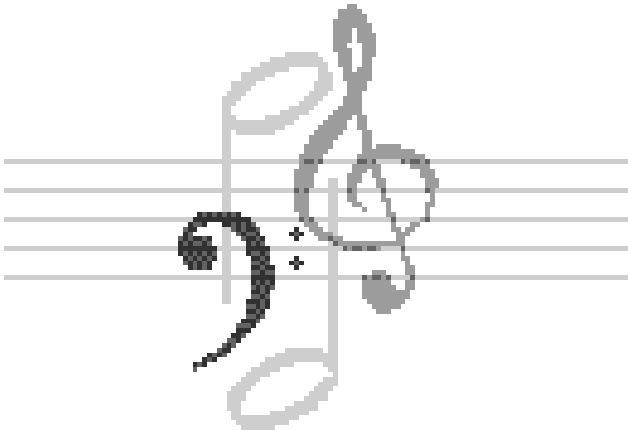


Tabel ANAVA

SV	JK	Db	RK	Fobs	Ftabel
Regresi	JKR	1			
Sesatan Tuna Cocok Sesatan Murni	JK(STC) JK(SM)	k-2 n-k	RK(STC) RK(SM)	RK(STC)/RK(SM)	Fk-2,n-k,alpha
Total	JKT	n-1			

$$\mathrm{JK}_{\mathrm{STC}} = \mathrm{JK}_{\mathrm{S}} - \mathrm{JK}_{\mathrm{SM}}, \quad \mathrm{JK}_{\mathrm{SM}} = \sum_{i,j} Y_{ij}^2 - \sum_{i} \frac{T_i^2}{n_i}$$

T_i : jumlah Y yang bersesuaian dengan X



Contoh yll

1. Susun Hipotesis :

H0: Hubungan X dan Y linier

H1: Hubungan X dan Y tidak linier

2. Alpha =0.05

Kelompok	X	Y	Y^2	n	Ti	Ti^2/ni
1	45	69	4761	2	140	9800
	45	71	5041			
2	50	71	5041	4	296	21904
	50	80	6400			
	50	76	5776			
	50	69	4761			
3	60	80	6400	3	254	21505.33
	60	85	7225			
	60	89	7921			
4	65	82	6724	3	261	22707
	65	93	8649			
	65	86	7396			
	665	951	76095			75916.33

Kelompok	X	Y	Y^2	n	Ti	Ti^2/ni
1	45	69	4761	2	140	9800
	45	71	5041			
2	50	71	5041	4	296	21904
	50	80	6400			
	50	76	5776			
	50	69	4761			
3	60	80	6400	3	254	21505.33
	60	85	7225			
	60	89	7921			
4	65	82	6724	3	261	22707
	65	93	8649			
	65	86	7396			
	665	951	76095			75916.33

$$JK_R = 541.693$$

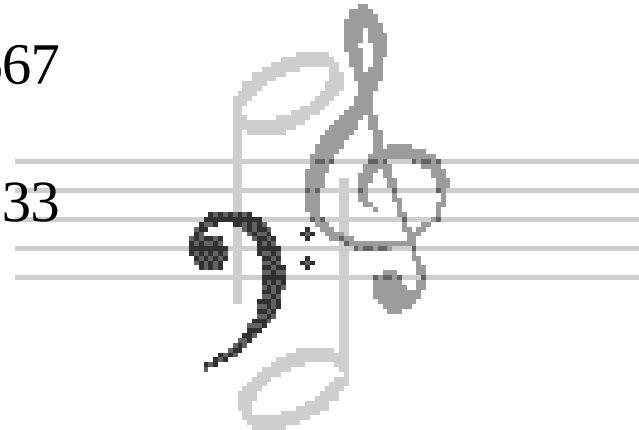
$$JK_T = 728.25$$

$$JK_S = 186.557$$

$$JK_{SM} = \sum_{i,j} Y_{ij}^2 - \sum_i \frac{T_i^2}{n_i} = 76095 - 75916.33 = 178.6667$$

$$JK_{STC} = JK_S - JK_{SM} = 186.557 - 178.6667 = 7.890333$$

T_i : jumlah Y yang bersesuaian dengan X



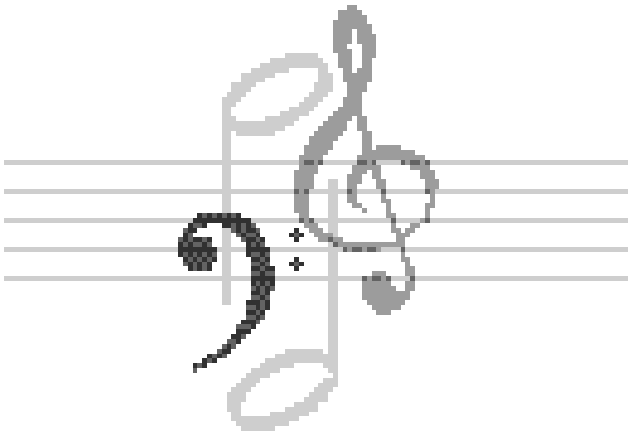
3. Tabel ANAVA

SV	JK	Db	RK	Fobs	Ftabel
Regresi	541.693	1			
Sesatan Tuna Cocok Sesatan Murni	7.890333 178.6667	4-2=2 12-4=8	3.945167 22.3333	0.176652	F(2,8,0.05)=4.46
Total	728.25	12-1=11			

4. Kesimpulan :

H0 tidak ditolak, karena Fobs=0.176652<Ftabel=4.46

d.k.l hubungan X dan Y linier



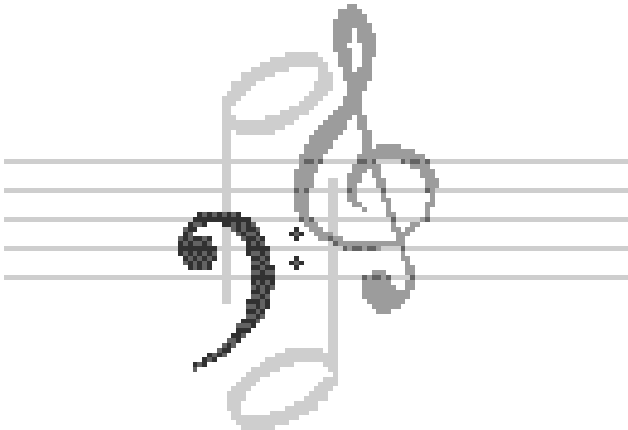
Uji Keberartian Regresi

1. Susun hipotesis

H_0 : Hubungan linier X dan Y tidak berarti

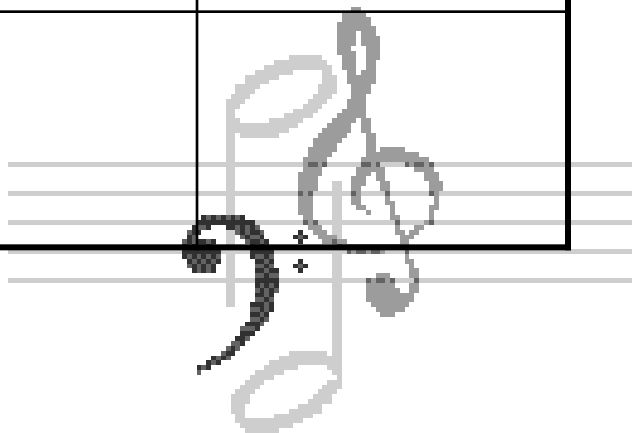
H_1 : Hubungan linier X dan Y berarti

2. Pilih tingkat signifikansi
3. Susun Anava
4. Kesimpulan : tolah Ho jika F> F tabel



Tabel Anava :

Sumber Variasi	JK	dk	RK	F Hitung
Regresi	JKR= $b^2 \sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2$	1	RKR=JKR/1	F=RKR/RKS
Sesatan	JKS= JKT-JKR	n-2	RKS=JKS/n-2	Ftabel F(alpha, 1,n-2)
Total	JKT= $\sum_{i=1}^{n} y_i^2 - \frac{\left(\sum y_i\right)^2}{n}$	n-1		



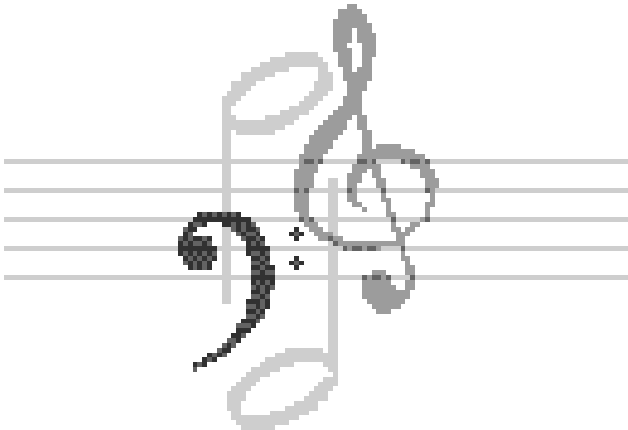
Contoh Yll

1. Susun hipotesis

H_0 : Hubungan linier X dan Y tidak berarti

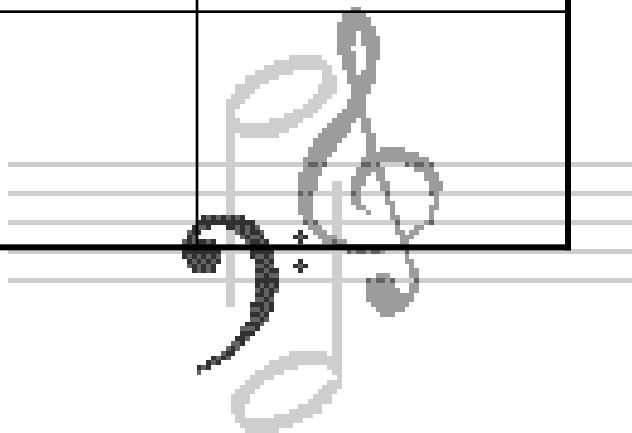
H_1 : Hubungan linier X dan Y berarti

2. Pilih tingkat signifikansi =0.05
3. Susun Anava
4. Kesimpulan : tolak Ho jika F> F tabel



Tabel Anava :

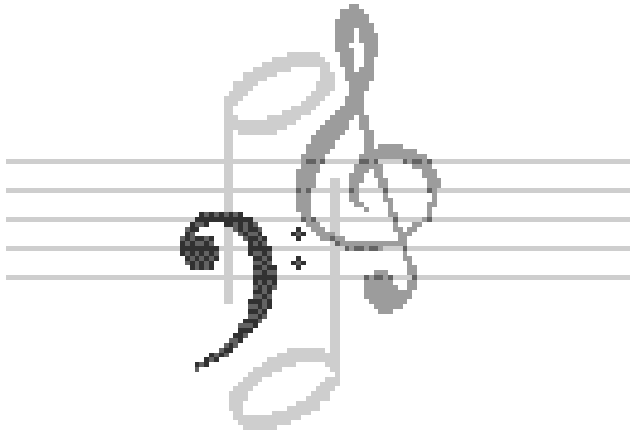
Sumber Variasi	JK	dk	RK	F Hitung
Regresi	541.193	1	541.193	29.04
Sesatan	186.557	12-2=10	18.6557	Ftabel F(alpha, 1,n-2)
Total	728.25	12-1=11		



4. Kesimpulan :

Tolak H0 karena Fobs=29.04>Ftabel=4.96

d.k.l regresi linier X dan Y berarti



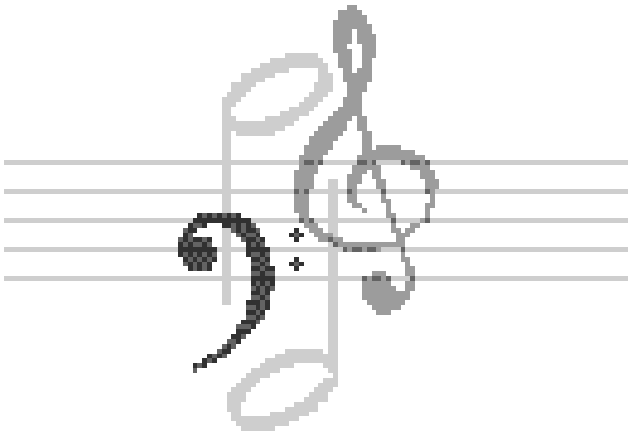
Uji Keberartian Koef. Regresi

1. Susun hipotesis

$$H_0 : \beta = 0$$
$$H_1 : \beta \neq 0$$

2. Pilih tingkat signifikansi
3. Kesimpulan : tolak Ho jika t> t tabel

$$t = \frac{b}{s_b}$$



Contoh Yll

1. Susun hipotesis

$$H_0 : \beta = 0$$
$$H_1 : \beta \neq 0$$

2. Pilih tingkat signifikansi α

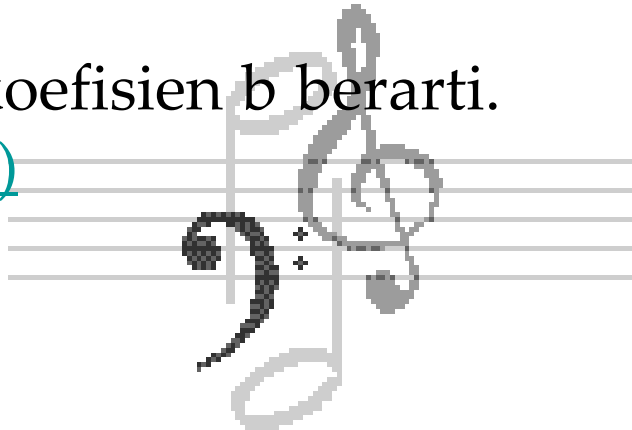
3. Kesimpulan : tolak Ho jika t> t tabel=t (α/2, n-2)

$$b = 0.8972$$

$$s_b = 0.166504$$

$$t = \frac{0.8972}{0.166504} = 5.388$$

Karena t=5.388>2.228 maka H0 ditolak jadi koefisien b berarti.
2.228 diperoleh dari tabel t dengan t(0.025,10)



Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.862[a]	.744	.718	4.319

a. Predictors: (Constant), matematik

ANOVA[b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	541.693	1	541.693	29.036	.000[a]
	Residual	186.557	10	18.656		
	Total	728.250	11			

a. Predictors: (Constant), matematik

b. Dependent Variable: fisika

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Collinearity Statistics	
		B	Std. Error	Beta			Tolerance	VIF
1	(Constant)	29.529	9.311		3.171	.010		
	matematik	.897	.167	.862	5.389	.000	1.000	1.000

a. Dependent Variable: fisika