

UJI UNTUK DISTRIBUSI NORMAL

Th.12.3.1 $X \sim N(\mu, \sigma^2)$, σ^2 diketahui

$$H_0 : \mu \leq \mu_0$$

$$H_1 : \mu > \mu_0$$

Pada tingkat signifikansi α , uji akan menolak H_0 jika

$$Z_0 \geq Z_{1-\alpha}$$

$$Z_0 = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \geq Z_{1-\alpha}$$

Jadi $\pi(\mu)$

$$\pi(\mu) = P\left[\frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \geq Z_{1-\alpha} \mid \mu \right]$$

$$= P\left[\frac{\bar{x} - \mu_0 + \mu - \mu}{\sigma / \sqrt{n}} \geq Z_{1-\alpha} \right]$$

$$= P\left[\frac{\bar{x} - \mu}{\sigma / \sqrt{n}} - \frac{\mu_0 - \mu}{\sigma / \sqrt{n}} \geq Z_{1-\alpha} \right] = P\left[\frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \geq Z_{1-\alpha} + \frac{\mu_0 - \mu}{\sigma / \sqrt{n}} \right] = 1 - \Phi\left[Z_{1-\alpha} + \frac{\mu_0 - \mu}{\sigma / \sqrt{n}} \right]$$

$X \sim N(\mu, \sigma^2)$, σ^2 diketahui

$$H_0 : \mu \geq \mu_0$$

$$H_1 : \mu < \mu_0$$

Pada tingkat signifikansi α , uji akan menolak H_0 jika

$$Z_0 \leq -Z_{1-\alpha}$$

$$Z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \geq -Z_{1-\alpha}$$

Jadi $\pi(\mu)$

$$\pi(\mu) = P\left[\frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \leq -Z_{1-\alpha} \mid \mu\right]$$

$$= P\left[\frac{\bar{x} - \mu_0 + \mu - \mu}{\sigma/\sqrt{n}} \geq -Z_{1-\alpha}\right]$$

$$= P\left[\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} - \frac{\mu_0 - \mu}{\sigma/\sqrt{n}} \geq -Z_{1-\alpha}\right] = P\left[\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \geq -Z_{1-\alpha} + \frac{\mu_0 - \mu}{\sigma/\sqrt{n}}\right] = 1 - \Phi\left[-Z_{1-\alpha} + \frac{\mu_0 - \mu}{\sigma/\sqrt{n}}\right]$$

Th. 12.3.2

$X \sim N(\mu, \sigma^2)$, σ^2 tidak diketahui

$$H_0 : \mu \leq \mu_0$$

$$H_1 : \mu > \mu_0$$

Pada tingkat signifikansi α , uji akan menolak H_0 jika

$$t_0 \geq t_{1-\alpha}(n-1)$$

$$t_0 = \frac{\bar{x} - \mu_0}{\sqrt{s/n}} \geq t_{1-\alpha}(n-1)$$

Jadi $\pi(\mu)$

$$\pi(\mu) = P\left[\frac{\bar{x} - \mu_0}{\sqrt{s/n}} \geq t_{1-\alpha}(v) \mid \mu \right], v = n-1$$

$$= P\left[\frac{\bar{x} - \mu_0 + \mu - \mu}{\sqrt{s/n}} \geq t_{1-\alpha}(v) \right]$$

$$= P\left[\frac{\bar{x} - \mu - (\mu - \mu_0)}{\sqrt{s/n}} \geq t_{1-\alpha}(v) \right] = P\left[\frac{Z + \delta}{\sqrt{V/v}} \geq t_{1-\alpha}(v) \right], \quad \delta = \frac{(\mu - \mu_0)}{\sqrt{s/n}}$$

$$X_i \sim N(\mu, \sigma^2)$$

$$\text{misalkan } v_0 = \frac{(n-1)s^2}{\sigma_0^2}$$

$$H_0 : \sigma^2 \leq \sigma_0^2$$

$$H_1 : \sigma^2 > \sigma_0^2$$

Pada tingkat signifikansi α , uji akan menolak H_0 jika

$$v_0 \geq \chi_{1-\alpha}^2 (n-1)$$

$$\pi(\sigma^2) = P[v_0 \geq \chi_{1-\alpha}^2 (n-1) | \sigma^2]$$

$$= P\left[\frac{(n-1)s^2}{\sigma_0^2} \geq \chi_{1-\alpha}^2 (n-1) | \sigma^2\right]$$

$$= P\left[\frac{(n-1)s^2}{\sigma_0^2} \geq \frac{\sigma^2}{\sigma^2} \chi_{1-\alpha}^2 (n-1)\right]$$

$$= P\left[\frac{(n-1)s^2}{\sigma^2} \geq \frac{\sigma_0^2}{\sigma^2} \chi_{1-\alpha}^2 (n-1)\right]$$

$$= 1 - H\left[\frac{\sigma_0^2}{\sigma^2} \chi_{1-\alpha}^2 (n-1)\right]$$

Th. 12.3.3 Uji untuk Variansi

Kerjakan soal
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Th. 12.3.4. Uji untuk dua sampel

$$X_i \sim N(\mu_1, \sigma_1^2), \quad Y_i \sim N(\mu_2, \sigma_2^2)$$

misalkan $f_0 = \frac{s_1^2}{s_2^2}$

$$H_0 : \frac{\sigma_2^2}{\sigma_1^2} \leq d_0$$

$$H_1 : \frac{\sigma_2^2}{\sigma_1^2} > d_0$$

Pada tingkat signifikansi α , uji akan menolak H_0 jika

$$f_0 \leq \frac{1}{f_{1-\alpha}(n_2 - 1, n_1 - 1)}$$

Pelajari juga Th. 12.3.5

12.4 Uji Binomial

$$X_i \sim BIN(1, p), \quad i = 1, \dots, n$$

$$\sum_{i=1}^n X_i \sim BIN(n, p)$$

Th. 12.4.1

1. $H_0 : p \leq p_0$

$$H_1 : p > p_0$$

Tolak H_0 jika

$$Z_0 = \frac{s - np_0}{\sqrt{np_0(1-p_0)}} > Z_{1-\alpha}$$

2. $H_0 : p \geq p_0$

$$H_1 : p < p_0$$

Tolak H_0 jika

$$Z_0 = \frac{s - np_0}{\sqrt{np_0(1-p_0)}} < -Z_{1-\alpha}$$

3. $H_0 : p = p_0$

$$H_1 : p \neq p_0$$

Tolak H_0 jika

$$Z_0 = \frac{s - np_0}{\sqrt{np_0(1-p_0)}} > Z_{1-\frac{\alpha}{2}} \quad \text{atau} \quad Z_0 = \frac{s - np_0}{\sqrt{np_0(1-p_0)}} < -Z_{1-\frac{\alpha}{2}}$$

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Uji POISSON

$$X_i \sim POI(\mu), \quad i = 1, \dots, n$$

$$S = \sum_{i=1}^n X_i$$

1. $H_0 : \mu \leq \mu_0$

$$H_1 : \mu > \mu_0$$

Tolak H_0 jika

$$1 - F(s-1, n\mu_0) \leq \alpha \quad \text{atau} \quad 2n\mu_0 \leq \chi^2_{1-\alpha}(2s)$$

2. $H_0 : \mu \geq \mu_0$

$$H_1 : \mu < \mu_0$$

Tolak H_0 jika

$$F(s-1, n\mu_0) \leq \alpha \quad \text{atau} \quad 2n\mu_0 \geq \chi^2_{1-\alpha}(2s)$$

3. $H_0 : \mu = \mu_0$

$$H_1 : \mu \neq \mu_0$$

Tolak H_0 jika

$$2n\mu_0 \geq \chi^2_{1-\frac{\alpha}{2}}(2s+2) \quad \text{atau} \quad 2n\mu_0 \leq \chi^2_{\frac{\alpha}{2}}(2s)$$

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MOST POWERFUL TEST

► Definisi 12.6.1

$$H_0 : \theta = \theta_0$$

$$H_1 : \theta = \theta_1$$

Berdasarkan C^* dapat dikatakan MPT dengan ukuran α jika

a. $\pi_{C^*}(\theta_0) = \alpha$

b. $\pi_{C^*}(\theta_1) \geq \pi_C(\theta_1)$

Lemma Neyman Person

$$X_i \sim f(x_1, \dots, x_n, \theta)$$

$$\lambda(x_1, \dots, x_n; \theta_0, \theta_1) = \frac{f(x_1, \dots, x_n; \theta_0)}{f(x_1, \dots, x_n; \theta_1)}$$

dan

$$C^* = \{(x_1, \dots, x_n) \mid \lambda(x_1, \dots, x_n; \theta_0, \theta_1) \leq k\}, k = \text{konstanta}$$

⇒

$$P[(x_1, \dots, x_n) \in C^* \mid \theta_0] = \alpha$$

Contoh soal

$$X_i \sim EXP(\theta)$$

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