BAB I

Bagian 3

Distribusi Kontinu

1. Continuous Uniform Distribution

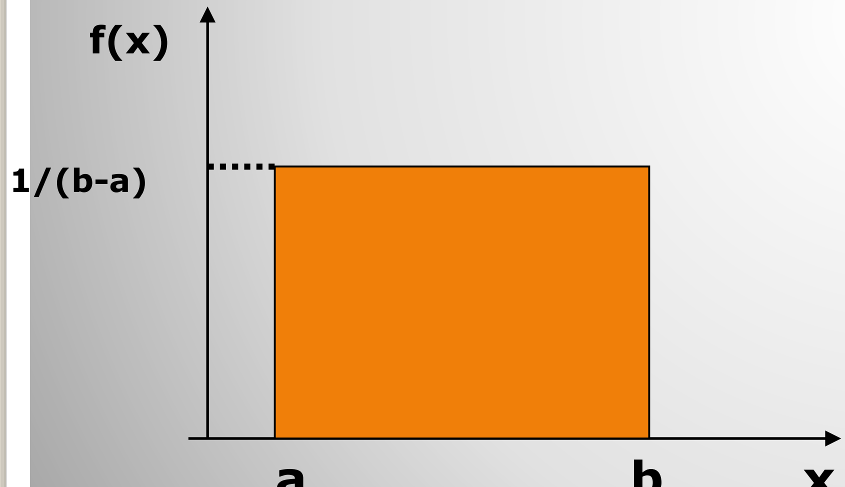
vR kontinu dikatakan mempunyai distribusi Uniform Kontinu X~UNIF(a,b) dengan pdf:

$$f(x;a,b) = \begin{cases} \dfrac{1}{b-a} & a \le x \le b \\ 0 & x \text{ yang lain} \end{cases}$$

Rata-rata :

$$\mu = \frac{a+b}{2}$$

Variansi : $\sigma^2 = \dfrac{(b-a)^2}{12}$



Cdf:

$$F(x) = \int_a^x 1/(b-a)\,du = x/(b-a) - a/(b-a)$$

$$F(x) = \begin{cases} 0 & x < a \\ (x-a)/(b-a) & a \le x < b \\ 1 & b \le x \end{cases}$$

contoh

Suatu vR kontinu menyatakan arus pada kabel dengan satuan mA dan mempunyai range [0,20mA], misalkan pdf dari X adalah

$$f(x) = 0.05, \quad 0 \le x \le 20$$

a. Berapa probabilitas ukuran arus antara 5 dan 10 mA?
b. Rata-rata dan variansi arus?

Penyelesaian:

$$P(5 < X < 10) = \int_5^{10} f(x)dx$$

$$= 5(0.05)$$

$$= 0.025$$

$$E(X) = 10 \quad V(X) = \frac{20^2}{12} = 33.33$$

2. Distribusi Gamma

Fungsi Gamma didefinisikan :

$$\Gamma(\kappa) = \int_0^\infty x^{\kappa-1} e^{-x}\, dx, \kappa > 0$$

Theorema

$$\Gamma(\kappa) = (\kappa - 1)\Gamma(\kappa - 1), \quad \kappa > 1$$

$$\Gamma(\kappa) = (\kappa - 1)!$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

Pdf dari Gamma:

$$X \sim GAM(\theta, \kappa), f(x; \theta, \kappa) = \frac{1}{\theta^\kappa \Gamma(\kappa)} x^{\kappa-1} e^{-\frac{x}{\theta}}, \quad x > 0$$

Rata-rata dan variansi

$$E[X] = \kappa\theta$$

$$V(X) = \kappa\theta^2$$

3. DISTRIBUSI NORMAL

- vR X dengan pdf :

$$f(x;\mu,\sigma)=\frac{1}{\sqrt{2\pi}\ \sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

$$\text{dengan parameter}\ \mu, \quad -\infty < \mu < \infty,$$

$$\text{dan}\ 0 < \sigma < \infty,$$

$$E(X)=\mu, \quad V(X)=\sigma^2$$

$$\mathrm{X} \sim N(\mu,\sigma^2)$$

Normal Standar PDF

$$\text{Jika}\, z = \frac{x-\mu}{\sigma} \quad \text{maka}\, \Phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, -\infty < z < \infty$$

$$Z \sim N(0,1)$$

$$E(X) = \mu = 0, \quad V(X) = \sigma^2 = 1$$

$$X \sim N(0,1)$$

→ϕ(z) maksimum saat z=0
→mempunyai titik belok saat z=±1

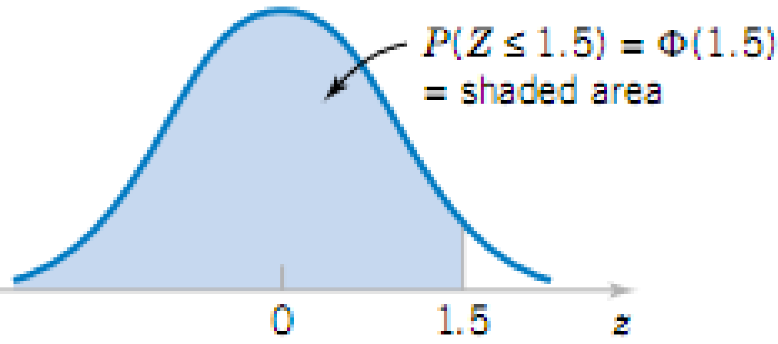
SIFAT-sifat

$$\Phi(z) = -\Phi(z)$$

$$\Phi'(z) = -z\Phi(z)$$

$$\Phi''(z) = (z^2 - 1)\Phi(z)$$

Contoh (Metstat 1)



z	0.00	0.01	0.02	0.03
0	0.50000	0.50399	0.50398	0.51197
⋮		⋮		
1.5	0.93319	0.93448	0.93574	0.93699

(1) $P(Z > 1.26) = 1 - P(Z \leq 1.26) = 1 - 0.89616 = 0.10384$

(2) $P(Z < -0.86) = 0.19490.$

(3) $P(Z > -1.37) = P(Z < 1.37) = 0.91465$

(4) $P(-1.25 < Z < 0.37).$

$$P(Z < 0.37) = 0.64431 \quad \text{and} \quad P(Z < -1.25) = 0.10565$$

Therefore,

$$P(-1.25 < Z < 0.37) = 0.64431 - 0.10565 = 0.53866$$

Theorema

Jika $X \sim N(\mu, \sigma^2)$ maka

1. $Z = \dfrac{X-\mu}{\sigma} \sim N(0,1)$

2. $F_X(x) = \Phi\left(\dfrac{x-\mu}{\sigma}\right)$

Pendekatan Normal dari Binomial

If X is a binomial random variable,

$$Z = \frac{X - np}{\sqrt{np(1-p)}} \qquad (4\text{-}12)$$

is approximately a standard normal random variable. The approximation is good for

$$np > 5 \quad \text{and} \quad n(1-p) > 5$$

Pendekatan Normal dari dist Poisson

If X is a Poisson random variable with $E(X) = \lambda$ and $V(X) = \lambda$,

$$Z = \frac{X - \lambda}{\sqrt{\lambda}} \tag{4-13}$$

is approximately a standard normal random variable. The approximation is good for

$$\lambda > 5$$

4. Distribusi Eksponential

vR X kontinu mempunyai distribusi Eksponensial dengan parameter $\theta > 0$ dan pdf :

$$f(x;\theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, \quad x > 0$$

therefore

$$F(x) = P(X \le x) = 1 - e^{-\lambda x}, \quad x \ge 0$$

and

$$f(x) = \lambda e^{-\lambda x}, \quad x \ge 0$$

$$E[X] = \theta$$

$$V(X) = \theta^2$$

Distribusi Lain

Kemungkinan tema yang bias diangkat dalam MK seminar (statmat):

- → Chi Kuadrat
- → Gumbell
- → Tweedie
- → Rayleight
- → Beta
- → Pearson
- → Cauchy
- → Benford
- → dll