Bab 3 Estimasi Interval

GENERAL METHOD

11.4 GENERAL METHOD

- **Jika kuantitas pivot tidak ada → masih mungkin menentukan IK untuk θ, jika statistik ada dengan distribusi yang tergantung θ tapi tidak parameter yang tidak diketahui**

Teorema 11.4.1. METODE UMUM

S Statistik yang kontinu dengan CDF $G(s;\theta)$,

anggap $h_1(\theta)$ dan $h_2(\theta)$ fungsi yang memenuhi

$G(h_1(\theta);\theta) = \alpha_1, \qquad 0 < \alpha_1 < 1$

$G(h_2(\theta);\theta) = 1 - \alpha_2, \quad 0 < \alpha_2 < 1$

jika $h_1(\theta)$ dan $h_2(\theta)$ fungsi naik dari θ maka memenuhi :

i. $h_2(\theta_L) = s$

ii. $h_1(\theta_U) = s$

iii. (θ_L, θ_U) IK untuk θ

jika $h_1(\theta)$ dan $h_2(\theta)$ fungsi turun dari θ maka memenuhi :

i. $h_2(\theta_U) = s$

ii. $h_1(\theta_L) = s$

iii. (θ_L, θ_U) IK untuk θ

Contoh soal

1. $X_i \sim EXP(\theta)$
2. $X_i \sim GAM(\theta, \mathrm{K})$

11.5 Interval Konfidensi untuk dua sampel

Flashback…

- Teorema 8.4.5 (Bain; 276)

Jika $X \sim F(\nu_1, \nu_2)$ maka $Y = \dfrac{1}{X} \sim F(\nu_2, \nu_1)$

dapat diketahui bahwa

$$\frac{1}{f_{1-\gamma}(\nu_1, \nu_2)} = f_\gamma(\nu_2, \nu_1)$$

$$f_{1-\gamma}(\nu_1, \nu_2) = \frac{1}{f_\gamma(\nu_2, \nu_1)}$$

Variansi

Theorema

$$\text{Jika } X_i \sim N\left(\mu_1, \sigma_1^2\right) \quad , \nu_1 = n_1 - 1 \quad \Rightarrow \nu_1 \frac{S_1^2}{\sigma_1^2} \sim \chi^2(\nu_1)$$

$$\text{Jika } Y_i \sim N\left(\mu_2, \sigma_2^2\right) \quad , \nu_2 = n_2 - 1 \quad \Rightarrow \nu_2 \frac{S_2^2}{\sigma_2^2} \sim \chi^2(\nu_2)$$

Dengan menggunakan Th 8.4.4, Bain:275

$$\left.\begin{aligned} V_1 &\sim \chi^2(\nu_1) \\ V_2 &\sim \chi^2(\nu_2) \end{aligned}\right\} \text{indep maka } X = \frac{\dfrac{V_1}{\nu_1}}{\dfrac{V_2}{\nu_2}} \sim F(\nu_1, \nu_2)$$

$$\text{Jika} \quad \left.\begin{array}{l} V_1 = \nu_1 \dfrac{S_1^2}{\sigma_1^2} \sim \chi^2(\nu_1) \\ V_2 = \nu_2 \dfrac{S_2^2}{\sigma_2^2} \sim \chi^2(\nu_2) \end{array}\right\} \text{independen} \Rightarrow \frac{\dfrac{V_1}{\nu_1}}{\dfrac{V_2}{\nu_2}} \sim F(\nu_1, \nu_2)$$

$$\frac{S_1^2}{S_2^2} \frac{\sigma_2^2}{\sigma_1^2} \sim F(\nu_1, \nu_2)$$

$$\frac{S_1^2}{S_2^2} \frac{\sigma_2^2}{\sigma_1^2} \sim F(n_1 - 1, n_2 - 1)$$

$$P\left(f_{\frac{\alpha}{2}}(\nu_1, \nu_2) < \frac{S_1^2}{S_2^2} \frac{\sigma_2^2}{\sigma_1^2} < f_{1-\frac{\alpha}{2}}(\nu_1, \nu_2) \right) = \gamma$$

$$P\left(f_{\frac{\alpha}{2}}(\nu_1, \nu_2) \frac{S_2^2}{S_1^2} < \frac{\sigma_2^2}{\sigma_1^2} < f_{1-\frac{\alpha}{2}}(\nu_1, \nu_2) \frac{S_2^2}{S_1^2} \right) = \gamma$$

Contoh 11.5.1

$n_1 = 16, S_1^2 = 0.6$

$n_2 = 21, S_2^2 = 0.2$

Tentukan IK 90% untuk $\dfrac{\sigma_2^2}{\sigma_1^2}$

IK UNTUK RERATA DUA SAMPEL

$$\overline{Y} - \overline{X} \sim N\left(\mu_2 - \mu_1, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$$

Jika σ_1^2, σ_2^2 diketahui, $\sigma_1^2 \neq \sigma_2^2$

$$Z = \frac{\overline{Y} - \overline{X} - (\mu_2 - \mu_1)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$$

$$P\left(-Z_{1-\frac{\alpha}{2}} < \frac{\overline{Y} - \overline{X} - (\mu_2 - \mu_1)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} < Z_{1-\frac{\alpha}{2}} \right) = \gamma$$

Jika σ_1^2, σ_2^2 diketahui, $\sigma_1^2 = \sigma_2^2$

Jika $T = \dfrac{\overline{Y} - \overline{X} - (\mu_2 - \mu_1)}{S_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$, $S_p = \dfrac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$

dan

$$V = \frac{(n_1 + n_2 - 2)S_p^2}{\sigma^2}$$

maka

$$\frac{(n_1 - 1)S_1^2}{\sigma_1^2} + \frac{(n_2 - 1)S_2^2}{\sigma_2^2} \sim \chi^2(n_1 + n_2 - 2)$$

Ingat!!

Jika

$$\left.\begin{aligned} Z &\sim \mathrm{N}(0,1) \\ V &\sim \chi^2(\nu) \end{aligned}\right\} Z \text{ dan V saling independen}$$

maka

$$\mathrm{T} = \frac{Z}{\sqrt{\dfrac{V}{\nu}}} \sim t(\nu)$$

Th. 8.4.1

Jika

$$\left.\begin{array}{l} Z \sim N(0,1) \\ V \sim \chi^2(\nu) \end{array}\right\} Z \text{ dan } V \text{ saling independen}$$

maka

$$T = \frac{Z}{\sqrt{\dfrac{V}{\nu}}} \sim t(\nu)$$

- Ingat dengan menggunakan th

$$Z = \frac{\bar{Y} - \bar{X} - (\mu_2 - \mu_1)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \sim N(0,1)$$

$$\frac{(n_1 - 1)S_1^2}{\sigma_1^2} + \frac{(n_2 - 1)S_2^2}{\sigma_2^2} \sim \chi^2(n_1 + n_2 - 2)$$

Dengan Th 8.4.1 diperoleh:

$$T = \frac{\dfrac{\bar{Y} - \bar{X} - (\mu_2 - \mu_1)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}}{\sqrt{\dfrac{\dfrac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{\sigma^2}}{n_1 + n_2 - 2}}} \sim t(n_1 + n_2 - 2)$$