UJI UNTUK DISTRIBUSI NORMAL

Th.12.3.1 $X \sim N(\mu, \sigma^2), \sigma^2$ diketahui

$\mathrm{H}_0 : \mu \leq \mu_0$

$H_1 : \mu > \mu_0$

Pada tingkat signifikansi α, uji akan menolak H_0 jika

$$\mathrm{Z}_0 \geq Z_{1-\alpha}$$

$$\mathrm{Z}_0 = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \geq Z_{1-\alpha}$$

Jadi $\pi(\mu)$

$$\pi(\mu) = P\left[\frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \geq Z_{1-\alpha} \middle| \mu\right]$$

$$= P\left[\frac{\bar{x} - \mu_0 + \mu - \mu}{\sigma / \sqrt{n}} \geq Z_{1-\alpha}\right]$$

$$= P\left[\frac{\bar{x} - \mu}{\sigma / \sqrt{n}} - \frac{\mu_0 - \mu}{\sigma / \sqrt{n}} \geq Z_{1-\alpha}\right] = P\left[\frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \geq Z_{1-\alpha} + \frac{\mu_0 - \mu}{\sigma / \sqrt{n}}\right] = 1 - \Phi\left[Z_{1-\alpha} + \frac{\mu_0 - \mu}{\sigma / \sqrt{n}}\right]$$

$X \sim N(\mu, \sigma^2), \sigma^2$ diketahui

$H_0 : \mu \geq \mu_0$

$H_1 : \mu < \mu_0$

Pada tingkat signifikansi α, uji akan menolak H_0 jika

$Z_0 \leq -Z_{1-\alpha}$

$$Z_0 = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \leq -Z_{1-\alpha}$$

Jadi $\pi(\mu)$

$$\pi(\mu) = P\left[\frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \leq -Z_{1-\alpha} \middle| \mu\right]$$

$$= P\left[\frac{\bar{x} - \mu_0 + \mu - \mu}{\sigma / \sqrt{n}} \leq -Z_{1-\alpha}\right]$$

$$= P\left[\frac{\bar{x} - \mu}{\sigma / \sqrt{n}} - \frac{\mu_0 - \mu}{\sigma / \sqrt{n}} \leq -Z_{1-\alpha}\right] = P\left[\frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \leq -Z_{1-\alpha} + \frac{\mu_0 - \mu}{\sigma / \sqrt{n}}\right] = \Phi\left[-Z_{1-\alpha} + \frac{\mu_0 - \mu}{\sigma / \sqrt{n}}\right]$$

Th. 12.3.2

$X \sim N(\mu, \sigma^2), \sigma^2$ tidak diketahui

$H_0 : \mu \le \mu_0$

$H_1 : \mu > \mu_0$

Pada tingkat signifikansi α, uji akan menolak H_0 jika

$$t_0 \ge t_{1-\alpha}(n-1)$$

$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \ge t_{1-\alpha}(n-1)$$

Jadi $\pi(\mu)$

$$\pi(\mu) = P\left[\frac{\bar{x} - \mu_0}{s/\sqrt{n}} \ge t_{1-\alpha}(v) \middle| \mu\right], v = n-1$$

$$= P\left[\frac{\bar{x} - \mu_0 + \mu - \mu}{s/\sqrt{n}} \ge t_{1-\alpha}(v)\right]$$

$$= P\left[\frac{\bar{x} - \mu - (\mu - \mu_0)}{s/\sqrt{n}} \ge t_{1-\alpha}(v)\right] = P\left[\frac{Z + \delta}{\sqrt{V/v}} \ge t_{1-\alpha}(v)\right], \quad \delta = \frac{(\mu - \mu_0)}{s/\sqrt{n}}$$

Th. 12.3.3 Uji untuk Variansi

$$X_i \sim N(\mu, \sigma^2)$$

$$\text{misalkan } v_0 = \frac{(n-1)s^2}{\sigma_0^2}$$

$$\mathrm{H}_0 : \sigma^2 \le \sigma_0^2$$

$$H_1 : \sigma^2 > \sigma_0^2$$

Pada tingkat signifikansi α, uji akan menolak H_0 jika

$$v_0 \ge \chi^2_{1-\alpha}(n-1)$$

$$\pi(\sigma^2) = P\left[v_0 \ge \chi^2_{1-\alpha}(n-1) \middle| \sigma^2\right]$$

$$= P\left[\frac{(n-1)s^2}{\sigma_0^2} \ge \chi^2_{1-\alpha}(n-1) \middle| \sigma^2\right]$$

$$= P\left[\frac{(n-1)s^2}{\sigma_0^2} \ge \frac{\sigma^2}{\sigma^2}\chi^2_{1-\alpha}(n-1)\right]$$

$$= P\left[\frac{(n-1)s^2}{\sigma^2} \ge \frac{\sigma_0^2}{\sigma^2}\chi^2_{1-\alpha}(n-1)\right]$$

$$= 1 - H\left[\frac{\sigma_0^2}{\sigma^2}\chi^2_{1-\alpha}(n-1)\right]$$

Kerjakan soal
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Th. 12.3.4. Uji untuk dua sampel

$X_i \sim N(\mu_1, \sigma_1^2), \quad Y_i \sim N(\mu_2, \sigma_2^2)$

misalkan $f_0 = \frac{s_1^2}{s_2^2}$

$H_0 : \frac{\sigma_2^2}{\sigma_1^2} \le d_0$

$H_1 : \frac{\sigma_2^2}{\sigma_1^2} > d_0$

Pada tingkat signifikansi α, uji akan menolak H_0 jika

$$f_0 \le \frac{1}{f_{1-\alpha}(n_2 - 1, n_1 - 1)}$$

Pelajari juga Th. 12.3.5

12.4 Uji Binomial

$$X_i \sim BIN(1, p), \quad i = 1, \ldots, n$$

$$\sum_{i=1}^{n} X_i \sim BIN(n, p)$$

Th.12.4.1

1. $\mathrm{H}_0 : p \le p_0$

   $\mathrm{H}_1 : p > p_0$

   Tolak H_0 jika

   $$Z_0 = \frac{s - np_0}{\sqrt{np_0(1 - p_0)}} > Z_{1-\alpha}$$

2. $\mathrm{H}_0 : p \ge p_0$

   $\mathrm{H}_1 : p < p_0$

   Tolak H_0 jika

   $$Z_0 = \frac{s - np_0}{\sqrt{np_0(1 - p_0)}} < -Z_{1-\alpha}$$

3. $\mathrm{H}_0 : p = p_0$

   $\mathrm{H}_1 : p \ne p_0$

   Tolak H_0 jika

   $$Z_0 = \frac{s - np_0}{\sqrt{np_0(1 - p_0)}} > Z_{1-\frac{\alpha}{2}} \quad \text{atau } Z_0 = \frac{s - np_0}{\sqrt{np_0(1 - p_0)}} < -Z_{1-\frac{\alpha}{2}}$$

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Uji POISSON

$$X_i \sim POI(\mu), \quad i = 1, \ldots, n$$

$$S = \sum_{i=1}^{n} X_i$$

1. $H_0 : \mu \leq \mu_0$

   $H_1 : \mu > \mu_0$

   Tolak H_0 jika

   $1 - F(s\text{-}1, n\mu_0) \leq \alpha$ atau $2n\mu_0 \leq \chi^2_{1-\alpha}(2s)$

2. $H_0 : \mu \geq \mu_0$

   $H_1 : \mu < \mu_0$

   Tolak H_0 jika

   $F(s\text{-}1, n\mu_0) \leq \alpha$ atau $2n\mu_0 \geq \chi^2_{1-\alpha}(2s)$

3. $H_0 : \mu = \mu_0$

   $H_1 : \mu \neq \mu_0$

   Tolak H_0 jika

   $2n\mu_0 \geq \chi^2_{1-\frac{\alpha}{2}}(2s+2)$ atau $2n\mu_0 \leq \chi^2_{\frac{\alpha}{2}}(2s)$

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MOST POWERFUL TEST

- Definisi 12.6.1

$$\mathrm{H}_0 : \theta = \theta_0$$

$$\mathrm{H}_1 : \theta = \theta_1$$

Berdasarkan $C*$ dapat dikatakan MPT dengan ukuran α jika

$$a.\ \pi_{C*}(\theta_0) = \alpha$$

$$b.\ \pi_{C*}(\theta_1) \geq \pi_{C}(\theta_1)$$

Lemma Neyman Person

$$X_i \sim f(x_1,\ldots,x_n,\boldsymbol{\theta})$$

$$\lambda(x_1,\ldots,x_n;\theta_0,\theta_1) = \frac{f(x_1,\ldots,x_n;\theta_0)}{f(x_1,\ldots,x_n;\theta_1)}$$

dan

$$C* = \{(x_1,\ldots,x_n) | \lambda(x_1,\ldots,x_n;\theta_0,\theta_1) \leq k\}, k = \text{konstanta}$$

$\ni$

$$P[(x_1,\ldots,x_n) \in C* | \theta_0] = \alpha$$

Contoh soal

$$X_i \sim EXP(\theta)$$

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