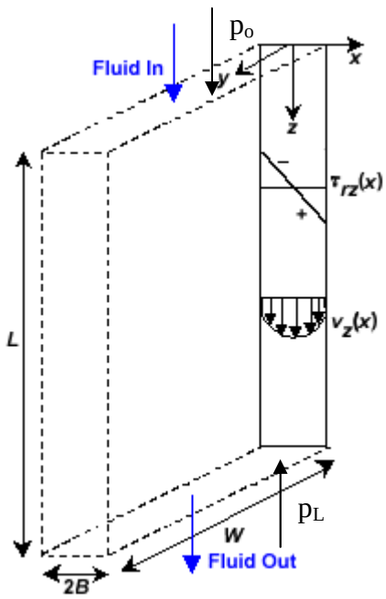
KUIS I
PROSES TRANSFER

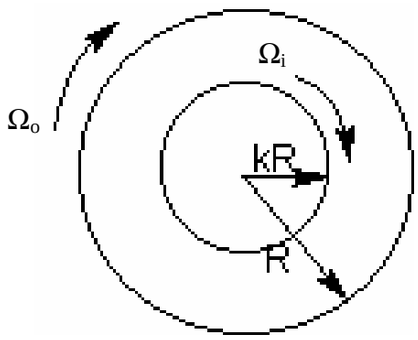
Waktu : 90 menit
Sifat : Tabel Terbuka

---

1. Fluida encer mengalir dengan aliran laminar pada celah yang terbentuk oleh 2 dinding parallel dengan jarak 2B. Fluida Newtonian dalam keadaan steady. Fluida mengalir karena perbedaan tekanan dan pengaruh gravitasi. Tentukan
   a. Fluks momentum
   b. Distribusi kecepatan
   c. Kecepatan rata – rata
   d. Debit (laju alir volum) fluida



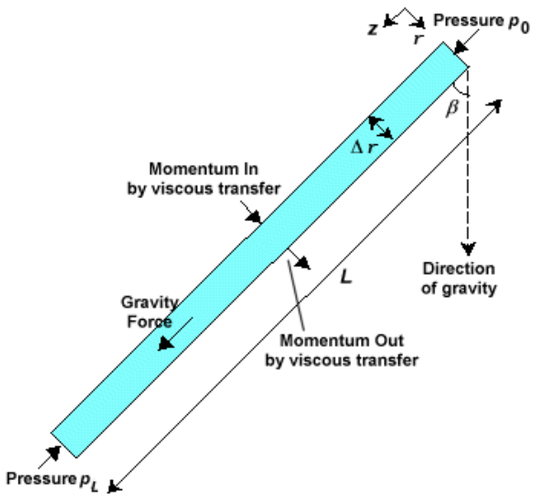
Gambar untuk soal no. 1



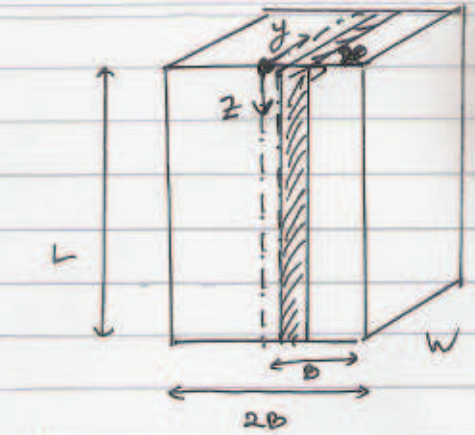
Gambar untuk soal no. 2

2. Tentukan $v_\theta(r)$ antara dua silinder koaksial dengan jari-jari R dan kR yang berputar pada kecepatan sudut Ω_o dan Ω_i. Anggap bahwa ruang antara 2 silinder diisi dengan fluida isotermal incompressible dalam aliran laminer.

3. Suatu fluida incompressible mengalir karena pengaruh perbedaan tekanan dan gaya gravitasi pada sebuah pipa dengan panjang L dan jari – jari R. Pipa membentuk sudut β dengan vertical. Tentukan distribusi kecepatan dan kecepatan rata – rata fluida.



1.



Kita hanya mengambil sebengah bagian saja. jadi titik nol ada tengah.
Syg kondisi batas adalah
pd $x = 0$; $\tau_{xz} = 0$ (di tengah)
pd $x = B$; $v_z = 0$ (di dinding dian

Neraca momentum pd elemen volume ($L.W.\Delta x$)

$$L.W\,\tau_{xz}\big|_x - L.W\,\tau_{xz}\big|_{x+\Delta x} + \rho\,\Delta x.W.v_z^2\big|_{z=0} + \rho\,\Delta x.W.v_z^2\big|_{z=L}$$

$$+ p_0\,\Delta x.W - p_L.\Delta x.W + L.W.\Delta x.\rho.g = 0$$

$$v_z|_{z=0} = v_z|_{z=L}$$

$$\lim_{\Delta x \to 0} \frac{\tau_{xz}|_{x+\Delta x} - \tau_{xz}|_x}{\Delta x} = \frac{(p_0 - p_L)}{L} + \rho.g$$

$$\frac{d\tau_{xz}}{dx} = \frac{\mathcal{P}_0 - \mathcal{P}_L}{L} \qquad \mathcal{P}_z = p_z - \rho.g.$$

$$\tau_{xz} = \frac{(\mathcal{P}_0 - \mathcal{P}_L)}{L} x + C_1$$

B.C. pd $x = 0$; v_z maks ; $\frac{dv_z}{dx} = 0$; $\tau_{xz} = 0$. $\Rightarrow C_1 = 0$

$$\boxed{\tau_{xz} = \frac{(\mathcal{P}_0 - \mathcal{P}_L)x}{L}}$$

$$-\mu \frac{dv_z}{dx} = \frac{(\mathcal{P}_0 - \mathcal{P}_L)x}{L}$$

$$\frac{dv_z}{dx} = -\frac{(\mathcal{P}_0 - \mathcal{P}_L)x}{\mu L}$$

$$v_z = -\frac{(\mathcal{P}_0 - \mathcal{P}_L)x^2}{2\mu L} + C_2$$

B.C. , pd $x = B$, $v_z = 0$ atau pd $x = -B$; $v_z = 0$.

$$C_2 = \frac{(\mathcal{P}_0 - \mathcal{P}_L)B^2}{2\mu L}$$

$$\boxed{v_z = \frac{(\mathcal{P}_0 - \mathcal{P}_L)B^2}{2\mu L}\left(1 - \frac{x^2}{B^2}\right)}$$

v_z maks pd $x = 0$ $\qquad v_z$ maks $= \frac{(\mathcal{P}_0 - \mathcal{P}_L)B^2}{2\mu L}$

$$\langle v_z \rangle = \frac{\int_0^W \int_0^B v_z \, dx \, dy}{W \cdot B}$$

$$= \frac{W \int \left(\frac{P_0 - P_L}{2\mu L}\right) \cdot B^2 \left(1 - \left(\frac{x}{B}\right)^2\right) dx}{W \cdot B}$$

$$= \frac{(P_0 - P_L)}{2\mu L} B^2 \int_0^B \left(1 - \left(\frac{x}{B}\right)^2\right) d\frac{x}{B}$$

$$= \frac{(P_0 - P_L)}{2\mu L} B^2 \left| \frac{x}{B} - \frac{1}{3}\left(\frac{x}{B}\right)^3 \right|_0^B$$

$$= \frac{(P_0 - P_L)}{2\mu L} B^2 \left(1 - \frac{1}{3}\right)$$

$$\boxed{\langle v_z \rangle = \frac{2}{3} \frac{(P_0 - P_L)}{2\mu L} B^2 = \frac{2}{3} v_{z\,max}}$$

$$Q = \langle v_z \rangle \cdot A$$

$$= \frac{2}{3} \frac{(P_0 - P_L)}{2\mu L} \cdot B^2 \cdot 2B \cdot W$$

$$\boxed{Q = \frac{2}{3} \frac{(P_0 - P_L)}{\mu L} B^3 W}$$

Tabel 3.4.1 – B

$$\frac{\partial \rho}{\partial t}^{S.S.} - \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)^{v_r = 0} + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z)^{v_z = 0} = 0$$

$$\frac{dv_\theta}{d\theta} = 0$$

Tabel 3.4.3 – E

$$\rho\left(\frac{\partial v_\theta}{\partial t}^{S.S.} + v_r \frac{\partial v_\theta}{\partial r}^{v_r = 0} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}^{0} + \frac{v_r v_\theta}{r}^{v_r = 0} + v_z \frac{\partial v_\theta}{\partial z}^{v_z = 0}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}^{p \text{ diabaikan}} +$$

$$\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}^{=0} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}^{v_r = 0} + \frac{\partial^2 v_\theta}{\partial z^2}^{v_\theta \neq z}\right] + \rho g_\theta^{g \text{ diabaikan}}$$

$$\mu \frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(r v_\theta)\right) = 0$$

$$\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(r v_\theta)\right) = 0$$

$$\frac{1}{r}\frac{d}{dr}(r v_\theta) = c_1$$

$$\frac{d}{dr}(r v_\theta) = c_1 r$$

$$r v_\theta = \frac{c_1 r^2}{2} + c_2$$

$$v_\theta = \frac{c_1 r}{2} + \frac{c_2}{r}$$

B.C. $r = kR \longrightarrow v_\theta = \Omega_i kR$

$r = R \longrightarrow v_\theta = \Omega_o R$

$$\Omega_i kR = \frac{C_1 kR}{2} + \frac{C_2}{kR} \quad \times \dots k \qquad \Omega_i k^2 R = \frac{C_1 k^2 R}{2} + \frac{C_2}{R}$$

$$\Omega_o R = \frac{C_1 R}{2} + \frac{C_2}{R} \longrightarrow \Omega_o R = \frac{C_1 R}{2} + \frac{C_2}{R}$$

$$R\left(\Omega_i k^2 - \Omega_o\right) = C_1\left(\frac{k^2 R}{2} - \frac{R}{2}\right)$$

$$C_1 = \frac{2\left(\Omega_i k^2 - \Omega_o\right)}{\left(k^2 - 1\right)}$$

$$C_2 = R\left(\Omega_o R - \frac{C_1 R}{2}\right)$$

$$= R\left(\Omega_o R - \frac{\left(2\Omega_i k^2 - 2\Omega_o\right)\frac{R}{2}}{k^2 - 1}\right)$$

$$= R\left(\frac{\Omega_o R k^2 - \Omega_o R - 2\Omega_i k^2 R + 2\Omega_o R}{k^2 - 1}\right)$$

$$= \frac{R^2 k^2}{k^2 - 1}\left(\Omega_o - \Omega_i\right)$$

$$\boxed{v_\theta = \frac{\left(\Omega_i k^2 - \Omega_o\right)}{\left(k^2 - 1\right)} \cdot r + \frac{R^2 k^2}{\left(k^2 - 1\right)}\left(\Omega_o - \Omega_i\right) \cdot \frac{1}{r}}$$

(3). Tabel 3.4.-1. B

$$\frac{dv_z}{dz} = 0$$

Soal ini sebenarnya sama dengan contoh soal aliran fluida dlm pipa. Perbedaan hanya pada $g \longrightarrow$ menjadi $g \cos\beta$.

Tabel 3.4-3. F

$$\rho\left(\frac{\partial v_z}{\partial t}^{S.S.} + v_r \frac{\partial v_z}{\partial r}^{v_r = 0} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}^{v_\theta = 0} + v_z \frac{\partial v_z}{\partial z}^{0}\right) = -\frac{\partial p}{\partial z}$$

$$+ \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}^{v_z \neq f(\theta)} + \frac{\partial^2 v_z}{\partial z^2}^{0}\right] + \rho g_z$$

$$0 = -\frac{\Delta P}{L} + \frac{\mu}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) + \rho g \cos\beta$$

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = +\frac{p_0 - p_L}{L} + \rho g \cos\beta$$

$$= +\frac{P_0 - P_L}{L} \qquad \text{dg } P_L = p_L + \rho g \cos\beta . L$$

$$\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = +\frac{\left(P_0 - P_L\right)}{\mu L} \cdot r$$

$$r\frac{dv_z}{dr} = -\frac{(P_0 - P_L)}{2\mu L} r^2 + C_1$$

$$\frac{dv_z}{dr} = -\frac{(P_0 - P_L)}{2\mu L} r + \frac{C_1}{r}$$

$$v_z = -\frac{(P_0 - P_L) r^2}{4\mu L} + C_1 \ln r + C_2$$

pd $r = 0$; v_z ada nilainya, $C_1 = 0$

$$v_z = -\frac{(P_0 - P_L) r^2}{4\mu L} + C_2$$

pd $r = R$; $v_z = 0$.

$$C_2 = \frac{(P_0 - P_L) R^2}{4\mu L}$$

$$v_z = \frac{(P_0 - P_L) R^2}{4\mu L}\left(1 - \left(\frac{r}{R}\right)^2\right)$$

$$v_{z\,maks} = \frac{(P_0 - P_L) R^2}{4\mu L}$$

$$\langle v_z \rangle = \frac{\int_0^{2\pi}\int_0^R v_z \cdot r\,dr\,d\theta}{\int_0^{2\pi}\int_0^R r\,dr\,d\theta}$$

$$= \frac{2\pi \int_0^R \frac{(P_0 - P_L) R^2}{4\mu L}\left(1 - \left(\frac{r}{R}\right)^2\right) r\,dr}{2\pi \cdot \frac{1}{2} R^2}$$

$$= \frac{2}{R^2}\int_0^R \frac{(P_0 - P_L) R^2}{4\mu L}\left(1 - \left(\frac{r}{R}\right)^2\right) r\,dr$$

$$= 2\frac{(P_0 - P_L) R^2}{4\mu L}\int_0^R \left(1 - \left(\frac{r}{R}\right)^2\right)\frac{r}{R}\,d\left(\frac{r}{R}\right)$$

$$= 2\frac{(P_0 - P_L) R^2}{4\mu L}\int_0^R \left[\frac{r}{R} - \left(\frac{r}{R}\right)^3\right] d\left(\frac{r}{R}\right)$$

$$= 2\frac{(P_0 - P_L) R^2}{4\mu L}\left[\frac{1}{2}\left(\frac{r}{R}\right)^2 - \frac{1}{4}\left(\frac{r}{R}\right)^4\right]_0^R$$

$$= 2\frac{(P_0 - P_L) R^2}{4\mu L}\left(\frac{1}{2} - \frac{1}{4}\right)$$

$$= \frac{1}{2}\frac{(P_0 - P_L) R^2}{4\mu L} = \frac{1}{2} v_{z\,maks}$$